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- Hoàng Nam Dũng (Hà Nội) Trajectory Optimization Problem (Bài toán tìm đường bay tối ưu trong hàng không)
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TÓM TẮT BÁO CÁO

Newton-type Schemes for Solving Generalized Equations

S. Adly¹, H. V. Ngai², and <u>N. V. Vu³</u>

Abstract: We consider a generalized equation between two Banach spaces X and Y of the form

find
$$x \in X$$
 such that $0 \in f(x) + F(x)$, (GE)

for a C^1 map $f: X \longrightarrow Y$ and a set-valued mapping $F: X \rightrightarrows Y$. Such a model covers a lot of applications in mathematics, engineering and sciences, e.g., the feasibility problem $(F(x) \equiv K \subset Y)$, variational inequality or complementarity problem (F(x) coincides with the normal cone to a closed convex set or closed convex cone, respectively). Because of its importance, the model (GE) has been studied widely by many authors. In this talk, we will discuss about some Newton-type schemes applied to (GE). The first section deals with the famous Josephy-Newton method and some new convergence results for such an algorithm. The second section is devoted to present another framework of solving (GE) for which both f and F are approximated. Our approaches are based on the basic tools from variational analysis, where the metric regularity property is a key point.

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Global Attractors for Nonclassical Diffusion Equations with Hereditary Memory and Exponential Nonlinearities

C. T. Anh¹, <u>D. T. P. Thanh²</u>, and N. D. Toan³

Abstract: In this talk, we study the existence and long-time behavior in terms of existence of global attractors of weak solutions to a class of nonclassical diffusion equations with hereditary memory and a new class of nonlinearities, which contains all nonlinearities of polynomial type, Sobolev type, and even exponential type. The main novelty of our result is that no restriction on the upper growth of the nonlinearity is imposed.

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Uniform Attractors for Nonlinear Viscoelastic Equations with Memory

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Abstract: We consider for $\rho \in [0, 1)$ and $\varepsilon > 0$, the following generalized Euler-Bernoulli plate equation with memory and singularly oscillating external force

$$u_{tt} + \Delta^2 u - \int_{-\infty}^t \mu(t-s)\Delta^2 u(s)ds + f(u) + g(u_t) = h_0(t) + \varepsilon^{-\rho} h_1(t/\varepsilon),$$

together with the averaged equation

$$u_{tt} + \Delta^2 u - \int_{-\infty}^t \mu(t-s)\Delta^2 u(s)ds + f(u) + g(u_t) = h_0(t)$$

formally corresponding to the limiting case $\varepsilon = 0$. Under suitable assumptions, the existence and uniform boundedness (w.r.t. ε) of the uniform attractors $\mathcal{A}^{\varepsilon}$ for the associated processes are established. We also prove the convergence of the attractor $\mathcal{A}^{\varepsilon}$ of the first equation to the attractor \mathcal{A}^{0} of the second one as $\varepsilon \to 0^{+}$.

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Study on the Qualitative Properties for Solutions of Equilibrium Problems and Related Problems

L. Q. Anh^1

Abstract: In this report we consider equilibrium problem and its extensions. We study some qualitative properties for solutions to such problems, including semicontinuity properties in the sense of Berge and Hausdorff, the Hölder/Lipschitz continuity of solution mappings and the well-posedness of approximate solutions in the sense of Hadamard, Tikhonov and Levitin-Polyak. Some efficient approaches to studying the stability conditions such as gap functions, scalarization methods are also discussed.

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On Hölder Continuity of Solution Maps to Parametric Equilibrium Problems

L. Q. Anh¹ and <u>**P.**</u> **T. Duoc**²

Abstract: In this paper, we consider the parametric equilibrium problems in normed spaces. By using the effective gap function approach, we establish the Hölder continuity of the solution maps of parametric equilibrium problems. As applications, the Hölder continuity of the solution maps for optimization problems, variational inequalities are derived.

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Penalty and Gap Function Methods for Lexicographic Equilibrium Problems

L. Q. Anh¹, N. P. Duc², and <u>T. Q. Duy³</u>

Abstract: In this talk, we consider lexicographic vector equilibrium problems. We propose a penalty function method for solving such problems. We show that every penalty trajectory of the penalized lexicographic equilibrium problem tends to the solution of the original problem. Using the regularized gap function, we obtain an error bound result for such penalized problems.

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On the Stability of Solution Mappings for Parametric Symmetric Vector Equilibrium Problems with Moving Cones

L. Q. Anh¹ and <u>**D. V.** Hien²</u>

Abstract: In this paper, we consider the parametric symmetric vector equilibrium problems. Stability conditions of solution mappings for these proplems are investigated. By using scalarization method we establish sufficient conditions for the Hausdorff lower semicontinuity and upper semicontinuity of the solution mappings. Our results are new or improve the existing ones in the literature.

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Gap Functions and Error Bounds for Generalized Mixed Vector Quasiequilibrium Problems

L. Q. Anh^1 , <u>N. V. Hung</u>², and V. M. Tam³

Abstract: In this paper, we consider generalized mixed vector quasiequilibrium problems in infinite-dimensional spaces. Then, we introduce gap functions for such problems in terms of regularized gap functions by using the nonlinear scalarization method. We establish sufficient conditions for these regularized gap functions to be continuous. Finally, error bounds for these problems in terms of regularized gap functions are studied. Our obtained results are new and improve the existing ones in the literature.

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Stability for Parametric Primal and Dual Equilibrium Problems

L. Q. Anh¹ and $\underline{\text{T. N. Tam}}^2$

Abstract: In this article, we consider parametric primal and dual equilibrium problems in locally convex Hausdorff topological vector spaces. Suffcient conditions for the approximate solution maps to be continuous are established. The similar results are obtained for the approximate solution maps of parametric weak vector equilibrium problems. As applications of these results, the continuity of the approximate solutions maps for optimization problems, variational inequalities is derived.

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Stability of Periodically Switched Discrete-Time Linear Singular Systems

P. K. Anh¹ and <u>**P. T.** Linh²</u>

Abstract: In this report we present some necessary and sufficient conditions for the stability of periodically switched discrete-time linear index-1 singular system, shortly, PSSS. In particular, it is proved that, if at least one subsystem of a PSSS is asymptotically stable, then there is a switching rule, so that the whole system is also uniformly exponentially stable. Furthermore, for a periodically switched control system with no stable subsystems, there exist a switching rule and feedback matrices, such that the obtained PSSS is uniformly exponentially stable.

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Mann-extragradient Method for Unrelated Variational Inequalities and Fixed Point Problems

<u>P. N. Anh¹</u>, J. K. Kim², and N. X. Phuong³

Abstract: This paper proposes a new hybrid variant of Mann and extragradient iteration methods for finding a common solution of a system of unrelated variational inequalities and fixed point problems corresponding to different feasible domains in a real Hilbert space. We present an algorithmic scheme that combines the idea of the extragradient method and the Mann iteration method as a hybrid variant. Then, the iterative point is modified by projecting a given initial point on intersect of suitable convex sets to get a strong convergence property under certain assumptions by suitable choice parameters. Finally, a numerical example is developed to illustrate the behavior of the new algorithm.

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A Strongly Convergent Subgradient Extragradient-Halpern Method for Solving a Class of Bilevel and Bilevel Split Pseudomonotone Variational Inequality Problems

P. K. Anh¹, L. D. Muu², and <u>T. V. Anh³</u>

Abstract: In this talk, we investigate a bilevel variational inequality problem (BVIP) and a bilevel split variational inequality problem (BSVIP) involving a strongly monotone mapping in the upper-level problem and pseudomonotone mappings in the lower-level one. The strongly convergent algorithms for BVIP and BSVIP are proposed and analyzed. In particular, a problem of finding the minimum-norm solution of a split pseudomonotone variational inequality problem is also studied. As a consequence, we get a strongly convergent algorithm for finding the minimum-norm solution to the split feasibility problem, which requires only two projections at each iteration step.

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Iterative Methods for Zeros of a Monotone Variational Inclusion in Hilbert Spaces

N. Buong¹ and <u>P. T. T. Hoai</u>²

Abstract: In this paper, we introduce implicit and explicit iterative methods for finding a zero of a monotone variational inclusion in Hilbert spaces. As consequence, an improvement modification of an algorithm existing in literature is obtained. A numerical example is given for illustrating the effectiveness of our algorithm.

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Iterative Algorithms for Zeros of a Maximal Monotone Operator on Hilbert Spaces

N. Buong¹ and <u>N. D. Nguyen</u>²

Abstract: In this paper, for a maximal monotone operator on a real Hilbert space with a nonempty set of zeros, we define a new nonexpansive mapping, whose fixed point set coincides with the set of zeros. Then, we introduce two new strongly convergent explicit iterative algorithms for finding a zero, that solves a certain monotone variational inequality problem. As special cases, new generalized and contraction proximal point algorithms with a summable sequence of resolvent parameters are obtained. Two numerical examples are given for illustration.

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Farkas-type Results for Vector Functional Inequalities Involving Composite Functions

M. J. Canovas¹, N. Dinh², <u>D. H. Long^{3,4}</u>, and J. Parra¹

Abstract: The present work is focussed on the generalization of the well-known Farkas lemma to the context of vector-systems involving composite functions defined on locally convex Hausdorff topological vector spaces. The paper establishes stable asymptotic Farkas-type results under certain convexity assumptions among others, which constitutes a generalization to the vector framework of its counterpart result for scalar systems [1]. Moreover, a non-asymptotic approach to Farkas lemma is also provided under some qualification conditions (without convexity assumptions). In the context of vector systems, the immediate antecedents in [2] and [3] can be found as special cases of this work where a particular structure (involving indicator functions) of composite functions is tackled. The results are then applied to characterize weak solutions for vector composite optimization problems.

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Dáng điệu nghiệm của phương trình tích phân Volterra và một vài ứng dụng trong hệ động lực

$\underline{\mathbf{D}}.~\underline{\mathbf{D}}.~\underline{\mathbf{Ch}}\underline{\mathbf{\hat{a}}}\mathbf{u}^1,~\mathbf{P}.~\mathbf{T}.~\mathbf{Nh}\underline{\mathbf{\hat{a}}}\mathbf{n}^2,~\mathbf{v}\underline{\mathbf{\hat{a}}}$ N. T. L. \mathbf{Hurong}^3

Tóm tắt: Trong báo cáo này, chúng tôi sẽ trình bày các điều kiện đủ về sự tồn tại nghiệm, sự ổn định nghiệm của phương trình tích phân Volterra trong không gian Banach. Sau đó sẽ xây dựng các ví dụ minh họa cho các mô hình ứng dụng trong lý thuyết hệ động lực tuyến tính bị nhiễu.

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Computation of Graphical Derivative for a Class of Normal Cone Mappings under a Very Weak Condition

N. H. Chieu¹ and <u>L. V. Hien²</u>

Abstract: In this report, we first present a formula for exactly computing the graphical derivative of the normal cone mapping $N_{\Gamma} : \mathbb{R}^n \to \mathbb{R}^n$, $x \mapsto N_{\Gamma}(x)$, under the condition that $M_q(x) := q(x) - \Theta$ is metrically subregular at the reference point, where $\Gamma := \{x \in \mathbb{R}^n | q(x) \in \Theta\}$, with Θ is a nonempty polyhedral convex set in \mathbb{R}^m and $q : \mathbb{R}^n \to \mathbb{R}^m$ is a twice continuously differentiable mapping. Then, based on this formula, we propose formulae for computing the graphical derivative of solution mappings and present characterizations of the isolated calmness for a broad class of generalized equations. Finally, applying to optimization, we get a new result on the isolated calmness of stationary point mappings.

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Ekeland's Principle for Vector Equilibrium Problems

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Abstract: In this paper, the authors deal with bifunctions defined on complete metric spaces with values in locally convex spaces ordered by closed convex cones. The aim is to provide a vector version of Ekeland's theorem related to equilibrium problems by weakening the lower semi-continuity of the objective bimaps. Via the vector Ekeland's principle, existence results for vector equilibria are proved in both compact and noncompact domains. We deduce several existence theorems on solutions for vector equilibrium problems, which extend and improve the related known results.

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Linesearch Algorithms for Split Equilibrium Problems and Nonexpansive Mappings

B. V. Dinh¹, <u>D. X. Son²</u>, L. Jiao ³, and D. S. Kim⁴

Abstract: In this paper, we first propose a weak convergence algorithm, called the linesearch algorithm, for solving a split equilibrium problem and nonexpansive mapping (SEPNM) in real Hilbert spaces, in which the first bifunction is pseudomonotone with respect to its solution set, the second bifunction is monotone, and fixed point mappings are nonexpansive. In this algorithm, we combine the extragradient method incorporated with the Armijo linesearch rule for solving equilibrium problems and the Mann method for finding a fixed point of an nonexpansive mapping. We then combine the proposed algorithm with hybrid cutting technique to get a strong convergence algorithm for SEPNM. Special cases of these algorithms are also given.

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Duality for Robust Optimization Problems

 $\underline{\mathbf{N.~Dinh}}^1$ and D. H. $\mathbf{Long}^{2,3}$

Abstract: In this report we give complete characterizations of strong duality for the *robust* vector optimization problem of the model:

(RVP) WMin {
$$F(x) : x \in C, G_u(x) \in -S, \forall u \in \mathcal{U}$$
 },

where X, Y and Z be locally convex Hausdorff topological vector spaces, S be a nonempty convex cone in Z, and K be a nonempty pointed convex cone in Y with $\operatorname{int} K \neq \emptyset, \mathcal{U}$ is an uncertainty set, $F: X \to Y \cup \{+\infty_Y\}, G_u: X \to Z \cup \{+\infty_Z\}$ for all $u \in \mathcal{U}$, and $C \subset X$ be a nonempty subset.

Using the theory of conjugate duality of vector-valued functions, some Farkas-type results for the system associated to (RVP) are established. Thank to these, two dual problems for (RVP) are proposed and then characterizations of strong duality for (RVP) associated to these two dual problems are derived in general setting. Next, we show that these characterizations can be simplified considerably in case where the data of (RVP) are convex and closed in certain sense, for instance, when F is K-convex and K-star-lower-semicontinuous; G is S-convex and its graph is S-epi-closed. The results give rise to new strong duality results even for usual vector optimization problems (i.e., when the uncertainty set is a singleton).

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Some Presentations of Epigraph of Conjugate Mappings with Applications to Vector Equilibrium Problems

N. Dinh¹, <u>N. D. Dung</u>², and D. H. Long^{3,4}

Abstract: We consider the vector equilibrium problem of the model:

(VEP) Find $x_0 \in A$ such that $F(x_0, x) \notin -intK$, $\forall x \in A$,

where X, Y, Z are locally convex Hausdorff topological vector spaces, $F : X \times X \to Y$, $G : X \to Z$ are mappings such that $F(x, x) = 0_Y$ for all $x \in X$, $\emptyset \neq C \subset X$ is a convex set. Let $A := C \cap G^{-1}(-S)$.

Several representations of the epigraph of the conjugate mapping $(F+I_A)^*$ are established under some constraint qualification conditions of interior-type such as Slater one and some of its generalizations aiming to the case where intS = \emptyset . The mentioned representations give rise to some versions of vector Farkas lemma and in turn, these results are then applied to (VEP) to get optimality conditions for weak solutions of this vector equilibrium problems.

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On the Global Łojasiewicz Inequality for Polynomial Functions

<u>D. V. Doat</u>¹ and H. H. Vui²

Abstract: Let $f: \mathbb{R}^n \longrightarrow \mathbb{R}$ be a polynomial in *n* variables. We give a method to check whether there exist constants $c > 0, \alpha > 0$ and $\beta > 0$ such that the following global Lojasiewicz inequality holds

$$|f(x)|^{\alpha} + |f(x)|^{\beta} \ge c.dist(x, f^{-1}(0))$$

for all $x \in \mathbb{R}^n$. In the case n = 2, the Lojasiewicz exponents are computed explicitly.

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Some Stability Properties of Parametric Quadratically Constraint Nonconvex Quadratic Programs in Hilbert Spaces

V. V. $Dong^1$

Abstract: Stability of nonconvex quadratic programming problems under finitely many convex quadratic constraints in Hilbert spaces is investigated. We present several stability properties of the global solution map, and the continuity of the optimal value function, assuming that the problem data undergoes small perturbations

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Trajectory Optimization Problem

H. N. Dung¹

Abstract: A few hours before each flight takes place a dispatcher computes and submits a route between departure and arrival airports to Air Traffic Control. In order to maximize profit airline companies want to minimize the cost of the submitted route. It is exactly the goal of the trajectory planning problem, an optimization problem with many constraints. Although in recent decades, there are softwares designed to particularly solve this problem, which have been used by many large airline companies, these softwares still reveal many limitations. Trajectory optimization is a difficult problem because each of its special cases is already NP-hard. In addition, from the practical requirements of the airline companies, we need to find solutions for a route within the time limit of 1 minute.

In this talk we introduce the trajectory optimization problem and present the results obtained from the cooperation project between Zuse Institute Berlin and Lufthansa System, the market leader in softwares for trajectory planning. Our three years of collaboration has delivered software prototype VOLAR, which is planned to replace the current software in the coming years.

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Dynamic Behavior of a Stochastic Predator-Prey System Under Regime Switching

N. H. Du¹, N. T. Dieu², and <u>T. D. Tuong³</u>

Abstract: In this paper we deal with regime switching predator-prey models perturbed by white noise. We give a threshold by which we know whenever a switching prey-predator system is eventually extinct or permanent. We also give some numerical solutions to illustrate that under the regime switching, the permanence or extinction of the switching system may be very different from the dynamics in each fixed state.

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An Adaptive Finite Element Method to Solve the Laplace–Beltrami Equation on Spheres with Spherical Splines

<u>P. T. Duong¹</u> and L. Tung¹

Abstract: We prove a posteriori upper and lower bounds for the error estimates when solving the Laplace–Beltrami equation on the unit sphere by using the Galerkin method with spherical splines. Adaptive mesh refinements based on these a posteriori error estimates are used to reduce complexity and computational cost of the corresponding discrete problems. The theoretical results are corroborated by numerical experiments.

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Monte Carlo Methods for Discontinuous Diffusions in Stratified Media

N. T. Q. Giang¹

Abstract: Diffusion equation appears in many different domains for example in electroencephalography, in molecular dynamics or in electrical impedance tomography. The method usually used for the numerical resolution this type of equation is the finite element method but this method possesses some disavantages like sensitivity to the dimension of the domain. This difficulty motivates us to develop Monte Carlo methods for the resolution of this equation. We consider only the divergence form operators with a piecewise constant diffusion coefficient and we will treat all type of boundary conditions. The Monte Carlo methods for Laplace equation with Dirichlet condition

$$\begin{cases} -\Delta u = 0, \ x \in D\\ u(x) = g(x), \ x \in \partial D \end{cases}$$

is based on the simulation of Brownian Motion using an efficace method like walk on spheres method [3] and Feynman-Kac formula [2]

$$u(x_0) = \mathbb{E}_{x_0}(g(W_{\tau_D})),$$

where $\{W_t\}_{t\geq 0}$ is a Brownian Motion et τ_D is the first time exit from the domain D. Stochastic finite differences techniques to treat the interface conditions [4] will be developed to treat the different kinds of boundary conditions, especially Robin conditions. By combining these two techniques : WOS and stochatic finite differences, we build random walks which score computed along the walk gives us a biased estimator of the solution of the partial differential equation we consider [1]. We prove that the global bias is in general of order two with respect to the finite difference step.

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A New Simple Parallel Iteration Method for a Class of Variational Inequalities

<u>N. S. Ha¹</u>, N. Buong², and N. T. T. Thuy³

Abstract: In this report, we will talk about a new simple parallel iterative method to find a solution for variational inequalities over the set of common fixed points of an infinite family of nonexpansive mappings on real reflexive and strictly convex Banach spaces with a uniformly Gâteaux differentiable norm. Some numerical examples are also given to illustrate the effectiveness and superiority of the proposed algorithm.

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On Data Dependence of Stability Domains, Exponential Stability and Stability Radii for Implicit Linear Dynamic Equations

<u>N. T. Ha¹</u>, N. H. Du², and D. D. Thuan³

Abstract: This talk deals with some questions concerning the stability domains, the spectrum of matrix pairs, the exponential stability and its robustness measure for linear implicit dynamic equations of arbitrary index. First, some characterizations of the stability domains corresponding to a convergent sequence of time scales are derived. Then, we investigate how the spectrum of matrix pairs, the exponential stability and the stability radii for implicit dynamic equations depend on the equation data when the structured perturbations act on both the coefficient of derivative and the right-hand side. The paper can be considered as a continued and complementary part to a recent paper on stability radius for implicit dynamic equations [N.H. Du, D.D. Thuan and N.C. Liem, Stability radius of implicit dynamic equations with constant coefficients on time scales, Systems Control Lett. 60 (2011) 596-603].

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Cơ sở toán học của học máy

Ð. N. Hào 1

Tóm tắt: Một số khái niệm cơ bản nhất của học máy có giám sát (supervised machine learning) được tổng kết. Cách nhìn nhận học máy như một bài toán ngược (đặt không chỉnh) được nêu ra. Tiếp cận học máy qua bài toán đặt không chỉnh, các vấn đề liên quan đến số chiều của bài toán và giảm thiểu số chiều, tốc độ hội tụ của các phương pháp ... được đề cập.

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Backward Semi-Linear Parabolic Equations with Time-Dependent Coefficients and Locally Lipschitz Source

D. N. Hao¹, N. V. Duc², and <u>N. V. Thang²</u>

Abstract: Let *H* be a Hilbert space with the inner product $\langle \cdot, \cdot \rangle$ and the norm $\|\cdot\|$, A(t) a positive self-adjoint unbounded time-dependent operator on *H* and $\varepsilon > 0$. We establish stability estimates of Hölder type and propose a regularization method with error estimates of Hölder type for the ill-posed backward semi-linear parabolic equation

$$u_t + A(t)u = f(t, u(t)), \ 0 < t \le T$$

 $\|u(T) - \varphi\| \le \varepsilon,$

with the source function f satisfying a local Lipschitz condition.

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Full Waveform Inversion Using Graphic Processing Units

<u>D. H. Hien¹</u>, T. Q. Minh¹, and M. T. Lua¹

Abstract: Full waveform inversion (FWI) has been considered as the most difficult issue in exploration geophysics as it requires massively computational time. However the outputs of FWI are considered as the most significant contribution factor to the petroleum exploration including seismic imaging and exploration drilling because the real physical parameters of subsurface such as P velocity, S velocity or density inverted directly from surface reflected seismic wave will be used for many exploration and drilling purposes. Consequently, this topic has been attractive for many researchers to study both theoretical improvement and computational time enhancement to bring the FWI to the petroleum exploration industrial applications. Among of the FWI improvements the parallel computing algorithm with multi cores processing has been used as a sufficient tool for overcome the computational time limitation. But it is very costly to build and operate a cluster system for FWI.

The fast developments of graphic processing units (GPU) for computational purpose allow us to implement heavily and massively parallel computational problems because each GPU card consists of many processing cores (from the hundred to the thousands number of cores) even though each processing core's frequency is much smaller than CPU. So that it motivates us to develop and implement the GPU-FWI.

In this presentation, we will show the theoretical background and derivation of FWI as an inverse problem ant its solutions by GPU. The numerical and real data set results will be shown.

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Fisrt Asymptotic Approximations to a Solution of Singularly Perturbed Optimal Control Problem with Intersecting Solutions of Degenerate Problem

N. T. Hoai¹

Abstract: Using the direct scheme method, we construct formally the zero order asymptotic approximation to an optimal control and the first order approximation to an optimal trajectory of a singularly perturbed optimal control problem with a weakly controllable state equation, a cheap control, a not uniquely solvable degenerate state equation, fixed endpoints and with intersecting trajectories of the degenerate state equation corresponding to slow trajectories of the perturbed problem. Together with boundary-layer functions in the vicinities of both ends of the considered interval, the constructed asymptotics contains inner boundary-layer functions. We consider the following problem

$$\begin{split} P_{\varepsilon} \colon J_{\varepsilon}(u) &= \int_{0}^{T} [F(t,\varepsilon)x + S(t,\varepsilon)y + \frac{1}{2}\varepsilon R(t,\varepsilon)u^{2}] dt \to \min_{u}, \\ dx/dt &= A(t,\varepsilon)x + B(t,\varepsilon)y + \varepsilon C(t,\varepsilon)u + f(t,\varepsilon), \quad \varepsilon dy/dt = G(x,y,t) + \varepsilon D(t,\varepsilon)u + g(t,\varepsilon), \\ &\quad x(0,\varepsilon) = x^{0}, \quad x(T,\varepsilon) = x^{T}. \end{split}$$

Here $t \in [0,T]$, T > 0 is fixed; $\varepsilon \ge 0$ is a small parameter; $z = z(t,\varepsilon)$ is scalar function, z = (x, y, u)'; all functions are sufficiently smooth with respect to their arguments; R(t,0) > 0 for all $t \in [0,T]$; $B(t_1,0) \ne 0$. Under the assumptions

A₁. The degenerate problem 0 = G(x, y, t) + g(t, 0), dx/dt = A(t, 0)x + B(t, 0)y + f(t, 0)with the conditions $x(0, \varepsilon) = x^0, x(T, \varepsilon) = x^T$ has two pairs of solution $y = \overset{(1)}{y}(x, t), \overset{(1)}{x}(t)$ and $y = \overset{(2)}{y}(x, t), \overset{(2)}{x}(t)$ with $\overset{(1)}{x}(t)$ and $\overset{(2)}{x}(t)$ intersect each other at one point $t = t_1 \in (0, T).$

 $\mathbf{A}_{2}. \ G_{y}\begin{pmatrix} 1 \\ x \end{pmatrix} \begin{pmatrix} 1 \\ t \end{pmatrix}, \begin{pmatrix} 1 \\ y \end{pmatrix} \begin{pmatrix} 1 \\ x \end{pmatrix} \begin{pmatrix} 1 \\ t \end{pmatrix}, t) > 0, \ t \in [0, t_{1}], \ G_{y}\begin{pmatrix} 2 \\ x \end{pmatrix} \begin{pmatrix} 2 \\ t \end{pmatrix}, \begin{pmatrix} 2 \\ y \end{pmatrix} \begin{pmatrix} 2 \\ x \end{pmatrix} \begin{pmatrix} 2 \\ t \end{pmatrix}, t) < 0, \ t \in [t_{1}, T]$ we construct the approximation to the solution of the problem P_{ε} which is found in the composed form $z(t, \varepsilon) = \overset{(1)}{z} (t, \varepsilon)$ if $t \in [0, t_{1}]$ and $z(t, \varepsilon) = \overset{(2)}{z} (t, \varepsilon)$ if $t \in [t_{1}, T]$ in the form

$$\overset{(j)}{z}(t,\varepsilon) = \sum_{i\geq 0} \varepsilon^{i} [\overset{(j)}{\overline{z}_{i}}(t) + \overset{(j)}{\Pi_{i}} z(\tau_{j-1}) + \overset{(j)}{Q_{i}} z(\tau_{j})] = \sum_{i\geq 0} \overset{(j)}{\widetilde{z}_{i}} (t,\varepsilon).$$

Theorem. For sufficiently small ε , the solution $z_*(t,\varepsilon)$ of problem P_{ε} satisfies the following estimates

$$|x_*(t,\varepsilon) - \widetilde{x}_1(t,\varepsilon)| \le c\varepsilon^2, \ |y_*(t,\varepsilon) - \widetilde{y}_1(t,\varepsilon)| \le c\varepsilon^2, \ |u_*(t,\varepsilon) - \widetilde{u}_0(t,\varepsilon)| \le c\varepsilon, \ J_{\varepsilon}(\widetilde{u}_0) - J_{\varepsilon}(u_*) \le c\varepsilon^2$$

Reference

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Điều khiển tối ưu cho cực đại thời gian lưu trú trên không của máy bay trước khi hạ cánh

T. N. Hùng¹

Tóm tắt: Chúng ta xem quá trình bay của máy bay là quá trình khuếch tán. Quá trình khuếch tán là nghiệm của hệ phương trình ngẫu nhiên $dX_t = b(X_t)dt + \sigma(X_t)dW_t$. Chúng ta khảo sát mô hình điều khiển quá trình khuếch tán sau:

Trước tiên ta có chiến lược điều khiển là hàm $u(t; x_s, s \leq t)$, lấy giá trị trong tập U. Hàm $\varphi(x), x \in \partial G$ là hàm thời gian tại thời điểm τ_G^u (là thời điểm đầu tiên mà X_t^u rời khỏi miền G).

Cho r = 1, G = (0, c), và $U = \{0, 1\}$; Hệ số gia tốc $a(x, u) \equiv 1$ và hệ số vận tốc b(x, u) = u. Nếu chúng ta sử dụng điều khiển u = 0, quá trình điều khiển chỉ là quá trình Wiener tiêu chuẩn và với u = 1 đó là quá trình Wiener cộng chuyển động đều với vận tốc bằng 1. Chúng ta muốn cực đại thời gian mà quá trình trải qua trong khoảng G trước thời điểm đầu tiên thoát ra khỏi miền G, trừ chi phí điều khiển mà nó tỉ lệ 1/2 đơn vị thời gian cho điều khiển u = 1, và 0 cho điều khiển u = 0. Chúng ta không tốn thời gian tại thời điểm $\tau_{(0,c)}$ rời khỏi khoảng đó: $\varphi(0) = \varphi(c) = 0$. Hàm mục tiêu của chúng ta là:

$$E\left(\int_{0}^{\tau_{(0,c)}^{x,u}} \left(1 - \frac{1}{2}u(t; X_s^{x,u}, 0 \le s \le t)\right)dt\right) = v(x) = \max$$

Kết quả ta được, cho G = (0, 10).

Trước tiên ta so sánh biến điều khiển tối ưu $\hat{u}(x)$ của hai quá trình Wiener.

$$\hat{u}(x) = v'(x) = \begin{cases} -\frac{1}{2} + e^{4.722 - 2x} & 0 \le x \le 2.361 \\ -2x + 5.222 & 2.361 \le x \le 10 \end{cases}$$
$$\hat{u}(x) = v'(x) = -2x + 10$$

Ta tiếp tục so sánh hàm kỳ vọng thời gian của hai quá trình Wiener lưu trú trong khoảng (0,10) .

$$v(x) = \begin{cases} -\frac{x}{2} - \frac{e^{4.722} - 2x}{2} + \frac{e^{4.722}}{2} & 0 < x \le 2.361 \\ -x^2 + 5.222x + 47.78 & 2.361 \le x \le 10 \\ v(x) = x(10 - x) \end{cases}$$

Điều khiển tối ưu là hướng quá trình tới phải và thêm 1 vào hệ số vận tốc để tới bên trái điểm 2.361, và rời khỏi điểm này quá trình của chúng ta là quá trình Wiener tiêu chuẩn.

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A New Approach to the Design of Observers For a Class of Time-Delay Systems

D. C. Huong¹

Abstract: Consider the following time-delay system:

$$\dot{x}(t) = Ax(t) + A_d x(t-\tau) + Bu(t), \ t \ge 0,$$
 (1)

$$x(\theta) = \phi(\theta), \ \theta \in [-\tau, 0], \tag{2}$$

$$y(t) = Cx(t), \tag{3}$$

where $\phi(\theta)$ is a continuous initial function, $\tau > 0$ is a known constant time delay, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $y(t) \in \mathbb{R}^p$ is the measurement output vector, matrices A, A_d, B and C are constant and of appropriate dimensions.

In this report, I present a new method for deriving state transformations of a class of timedelay systems with multiple output of the form (1)-(3). The significance of this method is that such state transformations can be used to transform system (1)-(3) into new coordinates where all the time-delay terms in the system description are associated with the output and input only. Therefore, in the new coordinate system, a Luenberger-type state observer can be readily designed. Subsequently, of the three possible versions of the original state vector, namely, instantaneous, delayed, and a mixed of instantaneous and delayed, a state observer which estimates one of these versions can be obtained. This new finding allows us to design state observers for a wider class of time-delay systems. Conditions for the existence of such coordinate changes and an effective algorithm for computing them are provided in this report. A numerical example is given to illustrate the simplicity and effectiveness of the proposed method.

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Some Combinatorial Enumerations on Permutations Avoiding a Mixed Pattern Set

T. T. T. Huong¹

Abstract: A permutation π is called avoiding another permutation τ , τ is called an ordinary pattern (or pattern if there is no confusing), if π does not contain any subsequence order-isomorphic to τ . In the literature, one investigates permutations avoiding a set of patterns for many variant purposes such as revisiting classical sequences, generating efficiently combinatorial objects, and discovering new sequences encoded by pattern avoiding permutations. For that, many types for avoiding a given pattern permutation like barred patterns, dotted patterns, and hatted patterns are introduced. In this talk, we consider permutations avoiding a mixed pattern set of ordinary patterns and hatted patterns. We give some combinatorial enumerations for these permutations which shows their relations to generalized Fibonacci sequences, Fine sequence, and Pisot sequence. Note that the result for the Pisot sequence is in fact an answer for a question posed by Jean-Luc Baril in 2011.

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Sensitivity Analysis of an Optimization Problem under Total Perturbations

D. T. K. Huyen¹, J. -C. Yao², and N. D. Yen³

Abstract: We analyze the stability of the Karush-Kuhn-Tucker (KKT) point set map of a C^2 -smooth parametric optimization problem with one C^2 -smooth functional constraint under total perturbations by applying a coderivative analysis of composite constraint functions of Levy and Mordukhovich [*Math. Program.*, 99 (2004), pp. 311–327] and several related results. We not only give necessary and sufficient conditions for the local Lipschitz-like property of the KKT point set map, but also sufficient conditions for its Robinson stability. These results lead us to new insights into the preceding deep investigations of Levy and Mordukhovich in the above-cited paper and of Qui [*J. Optim. Theory Appl.*, 161 (2014), pp. 398–429; *J. Glob. Optim.*, 65 (2016), pp. 615–635], and allow us to revisit and extend several stability theorems on quadratic programming under a quadratic constraint.

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Finite-Dimensional Feedback Control of Reaction-Diffusion Equations via Inertial Manifolds and Applications

N. T. Huy¹, <u>B. X. Quang²</u>, and D. D. Thuan³

Abstract: The notion of inertial manifolds was introduced in 1985 by C. Foias, G.R. Sell and R. Temam. An inertial manifold is a (at least Lipschitz) smooth finite-dimensional manifold of the phase space which is positively invariant, attracts exponentially all orbits, and contains the global attractor. In this talk, using the inertial manifold theory for evolution equations, we will construct a feedback controller for a class of one-dimensional nonlinear reaction-diffusion equation. We then apply the result to some evolutionary models.

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Inverse 1-Center Problem on Interval Graphs

N. T. Kien¹

Abstract: We address the problem of modifying weights of intervals in an interval graph at a minimum way of costs such that the predetermined interval become a 1-center of the perturbed graph. This problem is called inverse 1-center problem on interval graphs. To solve this problem, we first investigate the optimality criterion for an interval to be a 1-center. We prove that the 1-center function is indeed quasi-convex in the feasible set of intervals. Based on this property, we develop an algorithm that solves the problem in quadratic time.

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Inverse Fractional Knapsack Problems with Profits and Costs Modification

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Abstract: We address in this paper the problem of modifying both profits and costs of a fractional knapsack problem optimally such that a prespectified solution becomes an optimal solution with prespect to new parameters. This problem is called the inverse fractional Knapsack problem. Concerning the l_1 -norm, we first prove that the problem is NP-hard by reducing a Partition problem to it. The problem can be however solved in quadratic time if we only modify profit parameters. Additionally, we develop a quadratic time algorithm that solves the inverse fractional knapsack problem under l_{∞} -norm.

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Inverse 1-Center Problem on Trees under a Generalization of Chebyshev Norm and Bottleneck Hamming Distance

N. T. Kien¹ and $\underline{T. T. Le}^1$

Abstract: This talk is about the problem on modifying the edge lengths of a tree at minimum cost, w.r.t. a given objective function, such that the prespecified vertex become a 1-center in the perturbed tree. Especially, we consider the problem under an objective function which is a generalization of Chebyshev norm and bottleneck Hamming distance. Here, the problem under either Chebyshev norm or bottleneck Hamming distance was already investigated by Nguyen and Sepasian (2015). To solve the problem under the general case, we first give the optimality criterion for a vertex that is a 1-center of a tree. Then we apply binary search to narrow the objective interval that contains an optimal solution. Finally, we find the optimal solution of the problem by an advanced search. It is shown that the problem can be solved in $O(cn \log n)$ time, where c is the number of break-points in the subfunctions (of the objective) and n is the number of vertices in the tree.

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A Note on Second-order (KKT) Optimality Conditions for Smooth Vector Optimization Problems

D. S. Kim¹ and <u>N. V. Tuyen²</u>

Abstract: In this note, we are interested in second-order Karush–Kuhn–Tucker (KKT) optimality conditions for the following constrained vector optimization problem

$$\min f(x)$$
(VP)
s.t. $x \in Q_0 := \{x \in \mathbb{R}^n : g(x) \le 0\},$

where $f := (f_i), i \in I := \{1, \ldots, l\}$, and $g := (g_j), j \in J := \{1, \ldots, m\}$ are twice continuously differentiable vector-valued functions.

One of the first investigations to obtain second-order (KKT) optimality conditions for smooth vector optimization problems was carried out by Wang [5]. Then, Bigi and Castellani [1, 2] obtained some weak second-order (KKT) optimality conditions by introducing some types of the second-order regularity conditions. In 2004, Maeda [3] was the first to propose a Abadie regularity condition and established strong second-order (KKT) necessary conditions for $C^{1,1}$ vector optimization problems. Then, using the so-called generalized Abadie regularity condition, Rizvi and Nasser [4] obtained some second-order (KKT) necessary conditions for (VP). However, the main result of Rizvi and Nasser [4, Theorem 3.2] is not correct.

The aim of this note is to present some second-order (KKT) necessary optimality conditions for (VP), which modify the incorrect result in [4, Theorem 3.2].

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Numerical Solution for a Class of Structured Strangeness-Free Differential-Algebraic Equations by Linear Multistep Methods

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Abstract: It is known that when we apply a linear multistep method to nonlinear strangenessfree differential-algebraic equations (DAEs), the strict stability of the second characteristic polynomial is required for the method stability. In this report we present and analyze implicit and half-explicit linear multistep methods for a class of structured strangeness-free DAEs. By applying the methods to reformulated DAEs, the methods have the same convergent order and stability property as applied to ordinary differential equations. That is, the strict stability of the second characteristic polynomial is relaxed. Numerical experiments are given to confirm the theoretical results.

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An Application of Krasnoselskii's Fixed Point Theorem to Nonlocal Initial Problem for Implicit First Order Fuzzy Fractional Differential Systems under Caputo gH-Differentiability

<u>H. V. Long¹</u>, N. T. K. Son², and N. P. Dong³

Abstract: This paper is devoted to consider solvability of implicit first order fuzzy differential systems with nonlocal conditions under gH-differentiability. The lack of complete continuity of the associated integral operators, due to the implicit form of the equations, is overcome by using Krasnoselskii's fixed point theorem in semilinear Banach space. Moreover, a vectorial version of Krasnoselskii's theorem and the technique based on vector-valued metric and matrices having the spectral radius less than one are likely to allow the system nonlinearities to behave independently as much as possible. In addition, the connection between the support interval and the constants from the growth conditions is highlighted.

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On the Euler-Maruyama Approximation for Stochastic Differential Equations with Irregular Coefficients

<u>N. H. Long¹ and D. Taguchi²</u>

Abstract: In the recent paper [Ngo, H-L., and Taguchi, D.: Strong rate of convergence for the Euler- Maruyama approximation of stochastic differential equations with irregular coefficients. *Mathematics of Computation*, 85(300), 1793-1819 (2016)], it has been shown that for stochastic differential equations whose drift coefficient is one-sided Lipschitz continuous and diffusion coefficient is $(\alpha + \frac{1}{2})$ -Hölder continuous, the strong rate of convergence of its Euler-Maruyama approximation is of order α in L^1 -norm. In this talk we will introduce a new method, which allows us to prove the same strong rate for the Euler-Maruyama approximation of SDEs whose drift coefficient is not necessary one-sided Lipschitz. We also discuss the strong rate for stochastic differential equations whose drift coefficient is Hölder continuous.

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Strong Rate of Tamed Euler-Maruyama Approximation for Stochastic Differential Equations with Makovian Switching, under Hölder Continuous Conditions of Diffusion Coefficient

N. H. $Long^1$ and <u>L. D. $Trong^2$ </u>

Abstract: We study the strong rate of convergence of the tamed Euler-Maruyama approximation for stochastic differential equations with Markovian switching (SDEwMSs) with super linear drift and Hödler continuous diffusion coefficient

$$X_t = x_0 + \int_0^t b(\theta_s, X_s) ds + \int_0^t \sigma(\theta_s, X_s) dW_s, \quad x_0 \in \mathbb{R}, t \in [0, T],$$

where (θ_t) is a Makov chain taking values in a finite state space $S = \{1, 2, ..., N\}$ with generator $\Gamma = (\gamma_{ij})_{N \times N}$ defined by

$$P\left[\theta_{t+u} = j | \theta_t = i\right] = \begin{cases} \gamma_{ij}u + o(u) & \text{if } i \neq j \\ 1 + \gamma_{ii}u + o(u) & \text{if } i = j \end{cases}$$

and $(W_t)_{0 \le t \le T}$ is a standard Brownian motion defined on a filtered probability space $(\Omega, F, T_{t \ge 0}, P)$.

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On Some Generalized Polyhedral Convex Constructions

<u>N. N. Luan¹</u>, J. -C. Yao², and N. D. Yen³

Abstract: Generalized polyhedral convex sets, generalized polyhedral convex functions on locally convex Hausdorff topological vector spaces, and the related constructions such as sum of sets, sum of functions, directional derivative, infimal convolution, normal cone, conjugate function, subdifferential, are studied thoroughly in this paper. Among other things, we show how a generalized polyhedral convex set can be characterized via the finiteness of the number of its faces. In addition, it is proved that the infimal convolution of a generalized polyhedral convex function is a polyhedral convex function. The obtained results can be applied to scalar optimization problems described by generalized polyhedral convex functions.

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Linear Convergence of Projection Algorithms

<u>D. N. Minh</u>¹ and **P. M. Hung**²

Abstract: Projection algorithms are well known for theirs simplicity and flexibility in solving feasibility problems. They are particularly important in practice since softwares involving projection algorithms require minimal implementation and maintenance. In this work, we study linear convergence of several projection algorithms for systems of finitely many closed sets. The results complement contemporary research on the same topic.

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Inverse Function Theorems in Graded Fréchet Spaces for Multifunctions

<u>H. V. Ngai¹</u> and M. Théra²

Abstract: In this talk, we present some inverse function theorems and implicit function theorem for set-valued mappings between Fréchet spaces. The proof relies on Lebesgue's Dominated Convergence Theorem and on Ekeland's variational principle. An application to the existence of solutions of differential equations in Fréchet spaces with non-smooth data is given.

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Stability in the Extended Trust Region Subproblem

T. V. $Nghi^1$

Abstract: Our purpose in this talk is to investigate the stability and the Lipschitzian stability of the Karush-Kuhn-Tucker point set map in parametric extended trust region subproblems (ETRS), in which the trust region intersects a ball with a single linear inequality constraint. The special structure of ETRS allows one to have deeper and sharper results on stability of this problem.

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Existence of Set-valued Vector Equilibrium Problems via Ekeland's Variational Principle

D. N. \mathbf{Quy}^1 and $\mathbf{\underline{B. Q. Viec}}^2$

Abstract: In this paper, we establish a new version of Ekeland variational principle, which improves the related results by weakening the lower semi-continuity of the objective bimaps. Via Ekeland's principle, existence results are proved for set-valued equilibrium problems in both compact and noncompact domains. Many example are provided to highlight relations of our results to existing ones, including their advantages.

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Numerical Identification for Independent Coefficients in Elliptic PDEs

T. N. T. Quyen¹

Abstract: In this talk I would like to present the inverse problem of identifying simultaneously the diffusion matrix, source term and boundary condition in the Neumann boundary value problem for an elliptic partial differential equation from a measurement data, which is weaker than required of the exact state. A variational method based on energy functions with Tikhonov regularization is here proposed to treat the identification problem. We discretize the PDE with the finite elements and prove the convergence as well as analyse error bounds of this approach.

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On the Upper-semicontinuity of the Solution Map to a Parametric Boundary Control Problem

N. H. Son^1

Abstract: Let Ω be a bounded domain in \mathbb{R}^2 with the boundary Γ of class $C^{1,1}$. We consider the following parametric elliptic optimal control problem (P). Determine a control function $u \in L^2(\Gamma)$ and a corresponding state function $y \in H^1(\Omega) \cap C(\overline{\Omega})$, which minimize the cost function

$$F(y, u, \mu) = \int_{\Omega} L(x, y(x), \mu^{(1)}(x)) dx + \int_{\Gamma} l(x, y(x), u(x), \mu^{(2)}(x)) d\sigma_{Y}(x) dx + \int_{\Gamma} l(x, y(x), \mu^{(2)}(x) dx + \int_{\Gamma} l(x, y(x), \mu^{(2)}(x)) d\sigma_{Y}(x) dx + \int_{\Gamma} l(x, y(x), \mu^{(2)}(x) dx + \int_{\Gamma} l(x, y(x), \mu^{(2)}(x)) d\sigma_{Y}(x) dx + \int_{\Gamma} l(x, y(x), \mu^{(2)}(x)) d\sigma_{Y}(x) dx + \int_{\Gamma} l(x, \mu^{(2)}(x)) d\sigma_{Y}(x) dx + \int_{\Gamma} l(x, \mu^{(2)}(x) dx + \int_{\Gamma} l(x, \mu^{(2)}(x)) d\sigma_{Y}(x) dx + \int_{\Gamma} l(x, \mu^{(2)}(x) dx + \int_{\Gamma} l(x) dx + \int_{\Gamma} l(x, \mu^{(2)}$$

subject to

$$\begin{cases} Ay + f(x, y) = 0 & \text{in } \Omega, \\ \partial_{n_A} y = u + \lambda^{(1)} & \text{on } \Gamma, \end{cases}$$
$$a(x) \le g(x, y) + u(x) + \lambda^{(2)} \le b(x) \text{ a.e. } x \in \Gamma, \end{cases}$$

where $L: \Omega \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, $l: \Gamma \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, $f: \Omega \times \mathbb{R} \to \mathbb{R}$ and $g: \Gamma \times \mathbb{R} \to \mathbb{R}$ are functions, $a, b \in L^2(\Gamma)$, a(x) < b(x) for a.e. $x \in \Gamma$, $(\mu, \lambda) \in (L^{\infty}(\Omega) \times L^{\infty}(\Gamma)) \times (L^2(\Gamma))^2$ is a vector of parameters with $\mu = (\mu^{(1)}, \mu^{(2)})$ and $\lambda = (\lambda^{(1)}, \lambda^{(2)})$.

In this talk, we present solution stability of problem (P). By using the direct method and the first-order necessary optimality conditions, we obtain the upper semicontinuity and continuity of the solution map with respect to parameters.

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Stability Radius of Switched Linear Positive Systems

N. K. Son¹ and <u>L. V. Ngoc²</u>

Abstract: In this report, we study stability properties of switched linear positive systems, in continuos-time as well as in discrete-time settings. The stability radii of switched systems in some special cases have been calculated.

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Về giảm bậc của hệ cơ học với tương tác di động

<u>N. T. Son¹</u> và T. Stykel²

Tóm tắt: Hệ cơ học với tương tác di động là mô tả toán học cho nhiều tình huống trong kỹ thuật như di chuyển của xe trên cầu, hoạt động của trục khuỷu trong piston, cơ chế tiện các chi tiết máy, và robot lắp rắp. Mô phỏng hoạt động của những hệ này thường đòi hỏi phải giải những hệ phương trình vi phân cấp hai cỡ rất lớn sinh ra từ việc rời rạc hóa sử dụng phương pháp FEM. Để quá trình mô phỏng có thể diễn ra trên một máy tính cá nhân trong thời gian chấp nhận được, việc sử dụng một phương pháp giảm bậc hữu hiệu là không thể tránh khỏi. Những phương pháp giảm bậc của mô hình tiêu chuẩn khó có thể sử dụng thành công. Lý do là đối với những hệ này, vector tải thay đổi chứ không cố định như những hệ thông thường. Báo cáo sẽ trình bày một lời giải cho bài toán trên.

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Solution Existence for Quadratic Programs in Reflexive Banach Spaces

N. N. Tam^1

Abstract: In this talk, we consider quadratic programs in Banach spaces and prove sufficient conditions for the solution existence of quadratic programs under finitely many linear constraints in reflexive Banach spaces.

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Quasi - Equilibrium Problems and Fixed Point Theorems of L.S.C Mappings

N. X. Tan^1 and <u>N. Q. Hoa</u>²

Abstract: In this talk, we apply our new results on quasi – variational inequality problems to generalized quasi – equilibrium problems. Some sufficient conditions on the existence of solution are shown. In particular, we establish several results on the existence of solutions to fixed points of lower semi – continuous mappings without conditions on closedness of values. These results generalize some well – known fixed point theorems obtained by previous authors as F. E. Browder and Ky Fan, X. Wu, L. J. Lin, and Z. T. Yu, etc.

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Directionally Generalized Differentiation for Multifunctions and Its Applications

<u>V. D. Thinh¹</u> and T. D. Chuong²

Abstract: The aim of this work is twofold. First, we establish sum rules for the directionally coderivatives of multifunctions and intersection rules for the directionally limiting normal cones. Then, we apply the provided formulas to derive directionally necessary conditions for a set-valued optimization problem.

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Explicit Criteria for Exponential Stability of Functional Differential Equations^{*}

C. T. $Tinh^1$

Abstract: In this talk, we present some explicit criteria for exponential stability of nonlinear time-varying functional differential equations. The obtained results include some existing stability criteria in the literature as particular cases. Our approach is novel and can be applied to various classes of differential equations (functional differential equations of neutral type, Volterra integro-differential equations, coupled difference-differential equations, etc).

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Dynamics of Soliton Amplitudes of the Perturbed Nonlinear Schrödinger Equation

<u>H. T. Toan^{1,2}</u>, A. Peleg³, and N. M. Quan⁴

Abstract: We study the perturbed nonlinear Schrödinger model for achieving transmission stabilization and switching of N colliding soliton sequences in optical waveguides. We demonstrate that dynamics of soliton amplitudes in N sequence transmission can be described by a system of N ordinary differential equations (ODEs). We then study the stability and bifurcation analysis of the ODEs to obtain conditions on the physical parameters for robust transmission stabilization as well as on-off and off-on switching of M out of N soliton sequences with $1 \leq M < N$. The theoretical predictions for amplitude dynamics obtained from the system of N ODEs are verified by the numerical simulations of the system of N coupled nonlinear Schrödinger equations with $2 \leq N \leq 4$ with the split-step Fourier method.

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Một vài định lý Minimax cho hàm đa trị trên nửa dàn tôpô

<u>V. V. Trí¹</u>, N. X. Hải², và N. H. Quân³

Tóm tắt: Chúng tôi giới thiệu các kết quả nghiên cứu của mình về phát triển một số dạng mở rộng của định lý minimax cũng như một số vấn đề liên quan cho các ánh xạ đa trị, đó là, chúng tôi thiết lập vài điều kiện đủ để có đẳng thức dạng

 $\inf \bigcup_{y \in X} \sup \bigcup_{x \in X} F(x, y) = \sup \bigcup_{x \in X} \inf \bigcup_{y \in X} F(x, y)$

và sự tồn tại điểm yên ngựa cho hàm đa trị $F: X \times X \to 2^R \setminus \{\emptyset\}$. Trong đó, X là một nửa dàn tôpô, nghĩa là X là không gian tôpô có trang bị thứ tự bộ phận thỏa mỗi cặp phần tử $(x, y) \in X \times X$ bất kỳ đều có cận trên đúng sup $\{x, y\}$ đồng thời ánh xạ $(x, y) \mapsto \sup \{x, y\}$ là liên tục từ $X \times X$ vào X.

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Short-term Flood Forecasting with a Semi-parametric Regression Ensemble Model

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Abstract: Flood forecasting is very important research topic in disaster prevention and reduction. The characteristics of flood involves a rather complex systematic dynamics under the influence of different meteorological factors including linear and non-linear pattern. Recently there are lots of novel forecasting ways of improving the forecasting accuracy. This paper explores the potential and effect of the semi-parametric regression to model the flood water-level for forecasting the inundation of Mekong Delta in Vietnam. The semi-parametric regression technique is a combination of a parametric regression approach and a non-parametric regression method. In the process of model building, three linear regression, partial least squares solution and multi-recursive regression method. They are used to capture flood linear characteristics. The second part of this model is solved by a modified estimation of a smooth function. Furthermore three justified non-linear regression models based on artificial neural network are also able to arrest flood non-linear characteristics. They help us to smooth the model's non-parametric constituent easily and quickly. The last element is the model's error. The proposed model is shown by the following equation:

 $Y_i = \beta_0^T X_i + g(\theta_0^T X_i) + \varepsilon_i.$

Then the semi-parametric regression is used for ensemble model based on the principle component analysis technique. Flood water-level forecasting, with a lead time of one and more days, has been made using a selected sequence of past water-level values and some relevant factors observed at a specific location. Time-series analytical method is utilized to build the model. Empirical results obtained indicate that the prediction using the semi-parametric regression ensemble model is generally better than those obtained using the other models presented in this study in terms of the same evaluation measurements. Our findings reveal that the estimation power of the modern statistical model is reliable and auspicious. The proposed model here can be used as a promising alternative forecasting tool for flood to achieve greater forecasting accuracy and optimize prediction quality further.

Key words: semi-parametric regression ensemble model, stepwise multiple linear, partial least squares, multi-recursive regression, artificial neural network, estimation, smooth function, food, water-level, prediction, forecasting, principle component analysis.

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Phân tích tính toán dữ liệu lịch sử để dự báo thị trường chứng khoán Việt Nam

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Tóm tắt: Trong bối cảnh kinh tế Việt Nam đang hội nhập mạnh mẽ, thị trường chứng khoán là một kênh huy động vốn lớn, khai thông dòng vốn cho nền kinh tế. Thị trường chứng khoán là một thị trường quan trọng giúp chính phủ theo dõi và vận hành các chính sách tài khóa một cách linh hoạt nhằm ổn định kinh tế vĩ mô. Vì vậy những biến động tích cực hay tiêu cực của thị trường chứng khoán là vấn đề rất được các chuyên gia quan tâm. Với mục tiêu nâng hạng thị trường chứng khoán Việt Nam lên thị trường "mới nổi" đến năm 2020, rất nhiều các tập đoàn, tổng công ty nhà nước đã cổ phần hóa và lên sản trong giai đoạn này. Chính vì vậy, phân tích tính toán dữ liệu để dự báo thị trường chứng khoán giúp các chiến lược gia, các nhà hoạch định chính sách, các chuyên gia kinh tế và các nhà dầu tư có đánh giá nhanh, chính xác và tổng thể về xu hướng của thị trường.

Hiện nay có rất nhiều các bài báo, báo cáo hội thảo về dự báo thị trường chứng khoán nhưng đa số chỉ dừng lại ở nghiên cứu trên lý thuyết. Với mong muốn giúp các nhà đầu tư có thể nắm bắt xu hướng thị trường và các mã cổ phiếu một cách nhanh chóng và chính xác các tác giả đã xây dựng phần mềm dự báo thị trường chứng khoán. Phần mềm hỗ trợ theo dõi chỉ số chứng khoán Việt Nam (VN-Index, HNX-Index, UPCOM-Index) và gần 1200 mã cổ phiếu. Dữ liệu giao dịch được thu thập đầy đủ từ những ngày đầu của thị trường chứng khoán Việt Nam cho tới nay, do vậy cơ sở dữ liệu là vô cùng lớn với hàng triệu bản ghi nhằm đánh giá chính xác xu thế thị trường. Phương pháp phân tích tính toán của chúng tôi dựa trên sự tổng hợp của các chỉ báo kỹ thuật như MA (Moving Averages), RSI (Relative Strength Indicator), MACD (Moving Average Convergence-Divergence). Phần mềm có thể đưa ra các cảnh báo sớm như: "Mua", "Bán", "Theo dõi", "Quá bán", "Quá mua" phù hợp trong đầu tư tại thi trường chứng khoán Việt Nam. Đặc biệt, để giúp hỗ trợ các nhà đầu tư, phần mềm có chức năng "khuyến nghi", "khuyến nghị sớm", "tìm kiếm", "phân tích cơ bản" giúp nhà đầu tư có thể nhanh chóng lưa chon các công ty niêm vết có cơ bản tốt và đang trong xu hướng tăng trưởng. Các cảnh báo mua bán và khuyến nghi rất nhanh chóng, đơn giản và theo kịp xu hướng của thị trường. Bên cạnh thị trường chứng khoán, phần mềm cũng phân tích thị trường hàng hóa về các nông sản, kim loại, năng lượng,... giúp các hiệp hội, các công ty trong các ngành liên quan có thể nắm bắt nhanh được xu hướng giá cả hàng hóa trên thế giới để có quyết định xuất nhập khẩu một cách chủ động. Hiện tại phần mềm FeStock đã có gần 500 lượt tải xuống sử dụng thường xuyên trên điện thoại di động và có những phản hồi tích cực về phần mềm.

Trong thời gian tới, nhóm tác giả tiếp tục mở rộng ứng dụng phần mềm trên các thị trường chứng khoán Hoa Kỳ, thị trường vàng, ngoại hối. Đồng thời phát triển ứng dụng trên nền tảng Web và mobile như iOS, Windows Mobile, BlackBerry.

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Nonlocal Problem for Differential Complementarity Systems

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Abstract: This paper aims to give the sufficient conditions for the existence of solutions of differential complementarity systems (DCS) of the form:

$$\begin{cases} x'(t) = f(t, x(t), u(t)) & \text{for a.e. } t \in (a, b), \\ K \ni u(t) \perp G(t, x(t)) + F(u(t)) \in K^* & \text{for a.e. } t \in (a, b), \\ x(a) = Mx, \end{cases}$$
(1.1)

where

K be a closed convex cone in \mathbb{R}^m and K^* the dual cone of K;

- $f: [a, b] \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n;$
- $G: [a, b] \times \mathbb{R}^n \to \mathbb{R}^m$ and $F: \mathbb{R}^m \to \mathbb{R}^m$ are continuous mappings;
- $M: C([a,b]; \mathbb{R}^n) \to \mathbb{R}^n$ is a linear and bounded operator.

An early version of DCS is the *variational inequality of evolution* which was introduced by C. Henry as a class of differential inclusions known as projected differential inclusions.

DCS (1.1) represents a control system with the state function x, input function u and output function y(t) = G(t, x(t)) + F(u(t)). DCS are usually studied with the "consistent state" $x(a) = x_0$ (see, e.g. W. P. H. Heemels, Linear Complementarity Systems: A Study in Hybrid Dynamics, Ph.D. Thesis, Department of Electrical Engineering, Eindhoven University of Technology, 1999). In present paper, we replace the condition $x(a) = x_0$ by a more general, nonlocal condition x(a) = Mx. Our aim is to give the sufficient conditions that guarantee the existence of solutions of (1.1).

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An Iterative Algorithm for Strong Vector Equilibrium Problem

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Abstract: In this report, an iterative algorithm for strong vector equilibrium problem (SVEP) is studied. We consider an auxiliary problem (AP) for SVEP and the relationships between these two problems. We also investigate existence condition of AP and SVEP. And then, an iterative algorithm for SVEP is proposed. Furthermore, we obtain convergence of this iterative algorithm under suitable conditions.

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