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TÓM TẮT BÁO CÁO

Solvability and Numerical Solution of a Fourth Order Nonlinear Elliptic Boundary Value Problem

<u>D. Q. A¹</u> and T. H. Hai²

Abstract: In this paper we consider a boundary value problem for fourth order nonlinear elliptic equation, which models a bending plate on nonlinear elastic foundation. Differently from other authors, here we propose a novel approach to investigation of solvability and numerical solution of the problem. Namely, we reduce it to an operator equation for the right hand side function and under some easily verified conditions we have established the existence and uniqueness of a solution. We have also constructed an iterative method for the solution of the problem. The theoretical results are illustrated on several examples. Besides, we discuss a phenomena in numerical realization of the iterative method for the nonlinear problem.

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Existence Results and Numerical Solution of a Fourth Order Nonlinear Boundary Value Problem

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Abstract: In this presentation, we consider the following fourth-order two-point boundary value problem

$$x^{(4)}(t) - f(t, x(t), x'(t), x''(t), x'''(t)) = 0, \quad t \in (0, 1),$$
(1)

$$x(0) = x'(1) = 0; ax''(0) - bx'''(0) = 0; cx''(1) + dx'''(1) = 0,$$
(2)

which models the bending equilibrium of an extensible beam.

We prove the existence and uniqueness of a solution of the problem (1), (2). Also, we propose an iterative method leading the solution of this problem to a sequence of simple linear boundary value problems for second order equation, which are easily solved numerically and the convergence of the method is established. Some numerical examples demonstrate the efficiency of the method.

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Sự tồn tại duy nhất nghiệm và phương pháp lặp giải một bài toán biên phi tuyến cấp bốn đầy đủ

Đ. Q. Á¹ và <u>N. T. K. Quy</u>²

Tóm tắt: Trong báo cáo này, chúng tôi xét một bài toán biên đối với phương trình phi tuyến cấp bốn đầy đủ dạng

$$u^{(4)}(x) = f(x, u(x), u'(x), u''(x), u'''(x)), \quad 0 < x < 1,$$
(3)

$$u(0) = u(1) = u''(0) = u''(1) = 0,$$
(4)

trong đó f là hàm số liên tục.

Bài toán này mô tả sự uốn của dầm trên nền đàn hồi phi tuyến với hai đầu mút được gối tự do trong trạng thái cân bằng. Dạng đơn giản của phương trình (3), cụ thể là phương trình

$$u^{(4)}(x) = f(x, u(x), u''(x))$$

đã được nhiều tác giả nghiên cứu bằng nhiều phương pháp khác nhau, trong đó chủ yếu là đưa bài toán về phương trình tích phân đối với ẩn hàm u(x). Gần đây chúng tôi đã đề xuất và phát triển một cách tiếp cận khác, đó là đưa bài toán về phương trình toán tử đối với vế phải. Trong báo cáo này, chúng tôi phát triển phương pháp này bằng cách đưa bài toán (3), (4) về phương trình toán tử đối với $\varphi = f(x, u, u', u'', u''')$. Nhờ đó chúng tôi đã thiết lập được sự tồn tại và duy nhất nghiệm trong các điều kiện dễ kiểm tra và xây dựng phương pháp lặp giải bài toán trên. Nhiều thí dụ minh họa được đưa ra chứng tổ hiệu quả của phương pháp.

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Further Results on Differential Stability of Convex Optimization Problems

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Abstract: As a complement to a recent paper by An and Yen [An, D.T.V., Yen, N.D.: Differential Stability of Convex Optimization Problems under Inclusion Constraints. Appl. Anal. 94, 108–128 (2015)] on subdifferentials of the optimal value function in parametric convex programming under inclusion constraints and functional constraints, this paper studies the differential stability of convex optimization problems under a regularity condition of Aubin's type [Aubin, J.-P.: Optima and Equilibria. An Introduction to Nonlinear Analysis. Springer-New York (1998)]. By a suitable sum rule for convex subdifferentials, we obtain exact formulas for the subdifferential and singular subdifferential of the optimal value function. Illustrative examples and a detailed comparison of our results with those of the above mentioned paper are given.

Key Words. Parametric convex programming; optimal value function; subdifferential; singular subdifferential; Aubin's regularity condition.

AMS Subject Classifications: 49J53; 49Q12; 90C25; 90C31

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Stabilization of Navier-Stokes-Voigt Equations

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Abstract: We consider the Navier-Stokes-Voigt equations in a smooth bounded domain with the homogeneous Dirichlet boundary condition. First, when the external force is timeindependent, we study the existence and exponential stability of strong stationary solutions to the problem. Then we show that any unstable steady state can be exponentially stabilized by using either an internal feedback control with support large enough or a multiplicative Ito noise of sufficient intensity. Finally, we show that under action of fast oscillating-in-time external forces, there exists a unique time periodic solution and any weak solution to the Navier-Stokes-Voigt equations converges to this time periodic flow with exponential speed in time.

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Optimal Control of the Unsteady 3D Navier-Stokes-Voigt Equations

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Abstract: We consider two optimal control problems for the 3D Navier-Stokes-Voigt equations in bounded domains. The first one is to minimize a quadratic objective functional and the second one is to minimize the time needed to reach a desired state. We show the existence of optimal solutions, the first-order and second-order necessary optimality conditions, and the second-order sufficient optimality conditions. The second-order optimality conditions obtained in the paper seem to be optimal in the sense that the gap between them is minimal.

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Local Exact Controllability to Trajectories of the Magneto-micropolar Fluid Equations

C. T. Anh¹ and <u>V. M. Toi</u>²

Abstract: In this work we consider the exact controllability to trajectories of the magnetomicropolar fluid equations with distributed controls supported in small sets. We first establish a new Carleman inequality for the associated linearized system which leads to its null controllability. Then, combining the null controllability of the linearized system with an inverse mapping theorem, we deduce the local exact controllability to trajactories of the whole nonlinear problem.

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Stability and Well-posedness in Vector Equilibrium Problems Involving Lorentz Cone

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Abstract: In this report, we consider the vector equilibrium problems involving Lorentz cone in metric spaces. Sufficient conditions for the solution maps to such problems to be upper semicontinuous and closed are established. Moreover, we also study necessary and/or sufficient conditions for these problems to be well-posed at the reference point. We provide numerous examples to explain that all the imposed assumptions are essential. Applications the achieved results to the Lorentz variational inequality are also discussed.

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On the Tykhonov Well-posedness of Lexicographic Equilibrium Problems

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Abstract: In this paper, we consider the vector equilibrium problems involving lexicographic cone in Banach spaces. We introduce the new concepts of the Tykhonov well-posedness for such problems. The corresponding concepts of the Tykhonov well-posedness in the generalized sense are also proposed and studied. Some metric characterizations of well-posedness for such problems are given. As an application of the main results, several results on well-posedness for the class of lexicographic variational inequalities are derived.

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Stability and Sensitivity Analysis for Equilibrium Problems and Related Problems

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Abstract: In this report, we consider the class of equilibrium problem. Sufficient conditions for the stability and sensitivity analysis of the solution mappings of such problems are proposed. These topics are also studied for some generalized settings related to equilibrium problems.

Keywords: Equilibrium problems, variational inequality, optimization problems, variational inclusion, stability, sensitivity analysis

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On the Semicontinuity of Vector Mappings and Properties of the Solutions to Vector Quasiequilibrium Problems

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Abstract: In this paper, we study some important properties of the upper and lower semicontinuity involving moving cones of vector mappings. Using these generalized semicontinuous conditions together with some assumptions related to continuity property, we investigate the properties of the solutions to weak and strong vector quasiequilibrium problems. All kinds of properties are considered: the compactness of the solution sets, the upper semicontinuity of the solution mappings and the well-posedness for such problems. Many examples are provided to illustrate the essentialness of the imposed assumptions. As applications, we apply the main results to lower and upper bounded equilibrium problems and elastic traffic network problems.

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Painlevé-Kuratowski Convergence of the Solution Sets for Generalized Vector Quasiequilibrium Problems

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Abstract: In this paper, we study Painlevé-Kuratowski convergence of the solution sets with a sequence of mappings converging continuously for the generalized vector quasiequilibrium problems. By virtue of a sequence of gap functions based on the nonlinear scalarization function and a key assumption (H_h) , we establish the necessary and sufficient conditions for Painlevé-Kuratowski lower convergence and Painlevé-Kuratowski convergence. Moreover, we apply some results on Painlevé-Kuratowski convergence of the solution sets of the generalized vector quasi-equilibrium problems to the generalized set-valued vector quasi-variational inequality problems. Some examples are given for the illustration of our results.

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Hội thảo Tối ưu và Tính toán Khoa học lần thứ 14

A Projection-Fixed Point Method for a Class of Bilevel Variational Inequalities with Split Fixed Point Constraints

<u>**T. V. Anh**¹</u> and **L. D.** Muu^2

Abstract: We propose a method for solving bilevel split variational inequalities involving strongly monotone operators in the leader problems and nonexpansive mappings in the follower ones. The proposed method is a combination of the projection method for variational inequality and the Krasnoselskii-Mann scheme for fixed points of nonexpansive mappings. Strong convergence of the iterative process is proved. Special cases are considered.

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Phương pháp đa bước giải phương trình vi tích phân dị biến và ứng dụng

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Tóm tắt: Rất nhiều quá trình vật lý hay các hiện tượng tự nhiên khi mô hình hóa thì sẽ thu được là các phương trình, hệ phương trình vi tích phân không thể giải được đối với thành phần chính trong phương trình vi tích phân được hiểu là tích phân của vecto hàm số có chứa ẩn. Sự tồn tại và duy nhất của nghiệm vi phân liên tục của bài toán biên đối với loại phương trình này cần phải được nghiên cứu và khảo sát. Thậm chí trong trường hợp nếu các nghiệm của bài toán này tồn tại, duy nhất và đủ trơn thì cũng không thể chỉ ra được nghiệm dưới dạng tích phân. Với những bài toán này nếu chúng ta áp dụng các phương pháp số bình thường đã được áp dụng để giải các phương trình vi tích phân (giải được đối với thành phần chính) thì kết quả thu được sẽ là một hệ phương trình đại số tuyến tính (hoặc phi tuyến) mà hoặc là không có nghiệm hoặc là có vô số nghiệm. Ngay cả trong trường hợp tuyến tính thì các phương pháp rời rạc quen biết thường sinh ra các quá trình không ổn định. Phương trình tổng quát của loại này có dạng:

$$A(t)x'(t) + F(t, x(t)) + \int_{0}^{t} G(t, s, x(s))ds = f(t),$$

với các điều kiện ban đầu cho trước.

Trong bài toán xét đến hai trường hợp cụ thể hệ số A(t):

- 1. $A(t) = t^p$, p > 0, $t \in (0, M]$, $F(t, x) \equiv 0$, $x(M) = \xi$, với x(t) là hàm số cần tìm;
- 2. A(t) ma trận cỡ $(n \times n)$, F(t, x(t)) và G(t, s, x(s)) vecto hàm số *n*-chiều, $t \in [0, 1]$, ngoài ra det $A(t) \equiv 0$, với điều kiện ban đầu

$$x(0) = x_0$$

Trong trường hợp thứ nhất loại phương trình này được gọi là phương trình vi tích phân dị biến và có dạng:

$$t^{p}x^{'}(t) = \int_{0}^{t} f(x(s))ds, \ t \in [0, M]$$

với điều kiện

$$x(M) = \xi.$$

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A New Explicit Iteration Method for a Class of Variational Inequalities

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Abstract: In this report, we introduce a new simple explicit iterative algorithm to find a solution for variational inequalities over the set of common fixed points of an infinite family of nonexpansive mappings on real reflexive and strictly convex Banach spaces with a uniformly Gâteaux differentiable norm. A numerical example also is given for illustration.

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Regularization for the Problem of Finding a Solution of a System of Nonlinear Monotone Ill-Posed Equations in Banach Spaces

N. Buong¹, N. T. T. Thuy², and <u>T. T. Huong³</u>

Abstract: In this report, we present an operator method of regularization for the problem of finding a solution of a system of nonlinear ill-posed equations with a monotone hemicontinuous mapping and N inverse-strongly monotone mappings in Banach spaces. A regularization parameter choice is given, and convergence rate of regularized solutions is estimated.

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Variational Convergence of Bifunctions on Nonrectangular Domains and Applications

<u>H. T. H. Diem¹</u> and P. Q. Khanh²

Abstract: Variational convergence of extended-real-valued functions has been developed for half a century with many important applications. In 2009 Jofre and Wets considered variational convergence of finite-valued bifunctions defined on rectangles and considered its variational properties. Since then, there have been a number of contributions in this direction including application in optimization. However, quasivariational problems cannot be expressed in terms of such bifunctions on rectangles, because their constraint sets depend on the variables of the problems. The aim of this paper is to extend epi/hypo and lopsided convergence, the main kinds of variational convergence of bifunctions, to the case of finitevalued bifunctions defined on nonrectangular domains and apply them to quasivatiational models. Their basic characterizations are established. Variational properties such as saddle points, minsup points, sup-projections, etc, of bifunctions are shown to be preserved for the limit bifunctions when the bifunctions epi/hypo converge to these limits (possibly under some additional assumptions) and applied to approximations of quasiequilibrium problems. The obtained results are new and, in the special case of bifunctions defined on rectangles, they also improve some known results.

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Vector Farkas-type Results Characterizing Weak Solutions of Vector Optimization Problems

N. $Dinh^1$

Abstract: Let X, Y and Z be locally convex Hausdorff topological vector spaces and $\emptyset \neq C \subset X$. The spaces Y and Z are partially ordered by the two closed convex cones K and S, respectively. Let further $F: X \to Y \cup \{+\infty_Y\}, G: X \to Z \cup \{+\infty_Z\}, A = G^{-1}(-S)$, and I_A denote the indicator function of the feasible set A, that is, the function associating the zero vector of Y to any element of A and the greatest element of Y to any element of $X \setminus A$. We consider the vector optimization problem

(VOP) WMin{
$$F(x) : x \in C, G(x) \in -S$$
}.

Several representations of the K-epigraph of the conjugate of the vector function $F + I_A$, i.e., $epi_K(F + I_A)^*$, are established. These are used as key tools to get variant versions of vector Farkas lemma, which go back or extend the known ones in the literature when when taking Y = R. Characterizations of weak solutions of the vector optimization problem (VOP) are then derived.

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Robust Vector Farkas-Type Results with Applications to Robust Vector Optimization Problems

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Abstract: We consider the robust vector optimization problem of the model

(RVOP) WMin{ $F(x) : x \in C, G_u(x) \in -S, \forall u \in \mathcal{U}$ },

where X, Y and Z are locally convex Hausdorff topological vector spaces and Y and Z are partially ordered by the two closed convex cones K and S, respectively; \mathcal{U} is an <u>uncertainty</u> set; $F: X \to Y \cup \{+\infty_Y\}, G_u: X \to Z \cup \{+\infty_Z\}$ for all $u \in \mathcal{U}$, and $\emptyset \neq C \subset X$. Set $\overline{A} = \bigcap_{u \in \mathcal{U}} G_u^{-1}(-S).$

In this report, some representations of the K-epigraph of the conjugate of the vector function $F + I_A$, i.e., $\operatorname{epi}_K(F + I_A)^*$, are established under some convexity assumptions and qualification conditions. These are then used as key tools to get robust Farkas-type results for vector systems associated to the vector problem (RVOP), allowing to derive optimality conditions for this problem.

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On the Solution Existence of Convex Quadratic Programming Problems in Hilbert Spaces

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Abstract: We provide solution existence results for convex quadratic programming problems in Hilbert spaces, which the constraint set is defined by finitely many convex quadratic inequalities. In order to obtain our results, we shall use either the concept of Legendre form or the concept of finite rank operator. The existence results are established without requesting coercivity of objective function or compactness of constraint set.

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Hội thảo Tối ưu và Tính toán Khoa học lần thứ 14

Splitting Algorithms for a Class of Bilevel Equilibrium Problems Involving Nonexpansive Mappings

<u>**P.** M. Duc¹ and L. D. Muu^2 </u>

Abstract: We propose a splitting algorithm for solving strongly equilibrium problems over the intersection of a finite number of closed convex sets given as the fixed-point sets of nonexpansive mappings in real Hilbert spaces. The algorithm is a combination between the gradient method and the Mann-Krasnosel'skii iterative scheme, which allows that the projection can be computed onto each set separately rather than onto their intersection. Strong convergence is proved. Some special cases involving bilevel equilibrium problems with reverse strongly monotone variational inequality, monotone equilibrium constraints and maximal monotone inclusions are discussed. An illustrative example involving a system of integral equations is presented.

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Stochastic Stability of Nonlinear Discrete-time Markovian Jump Systems with Time-varying Delay and Partially Unknown Transition Rates

N. T. $Dung^1$

Abstract: This paper is concerned with the stochastic stability of a class of nonlinear discrete-time Markovian jump systems with interval time-varying delay and partially unknown transition probabilities. A new weighted summation inequality is first derived. We then employ the newly derived inequality to establish delay-dependent conditions which guarantee the stochastic stability of the system. These conditions are derived in terms of tractable matrix inequalities which can be computationally solved using various convex optimized algorithms. Numerical examples are provided to illustrate the effectiveness of the obtained results.

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Shortest Ordered Paths along a Sequence of Adjacent Faces of a Polytope

N. N. Hai¹, P. T. An², and <u>P. T. T. Huyen³</u>

Abstract: We present some analysis and geometric properties of shortest paths between two points in Euclidean spaces E. Given two points $a, b \in E$ and a sequence of line segments e_1, \ldots, e_k in E, a path that joins a to b and goes through e_1, \ldots, e_k in that order is called an ordered path. An ordered path that is shortest is called a shortest ordered path. We discuss the existence and uniqueness of shortest ordered paths and conditions for concatenation of two shortest ordered paths to be a shortest ordered path. We then focus on shortest paths between two points on polygons, sequences of adjacent triangles in 2 and 3 dimensionalspaces, respectively, especially on straightest paths on the sequences of adjacent triangles.

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Proper Orthogonal Decomposition and its Applications

D. N. Hao^1

Abstract: Proper orthogonal decomposition (POD) was developed by several peoples: D. D. Kosambi (1943), M. Loève (1945), K. Karhunen (1946), V. S. Pougachev (1953) and A. M. Obukhov (1954). POD provides a basis for the modal decomposition of an ensemble of functions, such as data obtained by experiments. POD provides an efficient approach to capturing the dominant components of an infinite-dimensional (or of a huge dimensional) process with only finite many, often surprisingly few, modes. POD has applications in various discipline such as, fluid dynamics, image processing, signal analysis, weather forecast, oceanography, partial differential equations, optimal control, reduced order models ...

This talk aims to introducing the notion of POD and some of its applications in optimal control of systems governed by partial differential equations.

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Low Rank Representations for Gram Matrices of Some Certain Nonnegative Polynomials and its Applications *

L. T. $Hieu^1$

Abstract: Non-negative polynomials play a fundamental role in optimization problems arising in control, for example, in signal processing. This is hence an interesting class of polynomials. However, determining the non-negativity of a polynomial is NP-hard in general. Since any polynomial with real coefficients which is nonnegative can be approximated by a sum of squares polynomial. Many problems over nonnegative polynomials are thus reduced to ones over sum of squares polynomials.

Any sum of squares polynomial can be expressed by a positive semidefinite matrix, which is called its Gram matrix. Convex optimization problems over non-negative/sum of squares polynomials can hence be solved by SDP solvers. However, the resulting Gram matrices derived by SDP solvers are usually full rank.

In this talk, we aim at presenting the idea of finding low-rank Gram matrices of certain nonnegative/sum of squares polynomials. This method is then applied to some low-pass filter design problems (with finite or infinite impulse response).

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New Algorithms for Finding the Whole Set of Nondominated Solutions of Multiobjective Discrete Optimization Problems

P. T. Hoai¹

Abstract: In this paper, we present two new algorithms for finding the whole set of nondominated solutions for general multiobjective discrete optimization problems. One of them is our new method based on a simple and straightforward approach in which all nondominated solutions are obtained by solving a sequence of singleobjective problems over suitable domains. The search regions are managed by using the variant concepts of polyblock and copolyblock in theory of monotonic optimization. The remaining algorithm is an improved version of the recent algorithm of Kirlik and Sayın [A new algorithm for generating all nondominated solutions of multiobjective discrete optimization problems. Eur. J. Oper. Res., 232: 479-488, 2014]. We show the efficiency of the new algorithms by testing on benchmarking examples in Kirlik and Sayın's paper and other randomly generated multiobjective knapsack and multiobjective assignment problems with the higher number of criteria as well as decision variables. Our new algorithm is verified as the best of running time among them.

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On Chi-Square Type Random Variables with Geometric Degrees of Freedom

T. L. Hung¹

Abstract: This paper deals with the distribution of chi-squared type random variables with geometric degrees of freedom. An explicit formula of density function is given and the asymptotic behavior of the limit distribution is shown as the parameter for the geometric distribution goes to zero.

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Variational Convergence and Applications in Optimization and Scientific Computing

P. Q. Khanh¹

Abstract: Convergence is the first basic notion in continuous mathematics which plays important roles in most areas of this "half" of mathematics. There have been many definitions of convergence in analysis, probability theory, numerical methods, etc. For optimization and related fields, naturally kinds of convergence which preserve variational properties like being minimum points, minsup points, saddle points, extremal values, etc, are crucial from various aspects. Variational convergence is the general terminology for such kinds of convergence.

Epi convergence for unifunctions, epi/hypo convergence and lopside convergence for bifunctions are main notions of variational convergence. Epi convergence was introduced more than half a century ago, and the other two have been developed for three decades now. In 2009, lopside convergence for finite-valued bifunctions was proposed and lead to a new effective approach for variational convergence and applications.

In this talk, we focus on lopside and epi/hypo convergence of finite valued bifunctions and applications in optimization. We also aim to a systematic exposition of the theory of variational convergence, from epi convergence of unifunctions to lopside and epi/hypo convergence and finally to applications in approximating optimization problems and estimating solutions of stochastic optimization problems. We present the theory for the general setting of topological spaces (based on a preprint of ours), but try to illustrate it by simple applications (taken from our several recent contributions).

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First- and Second-Order Optimality Conditions for Vector Nonsmooth Semidefinite Programming

P. Q. Khanh¹ and <u>**L. T. Tung**</u>²

Abstract: By using Fréchet pseudo-Jacobian and pseudo-Hessian for the vector-valued maps defined on the space of symmetric matrices, we establish the first and second-order optimality conditions for some kind of efficient solutions of vector nonsmooth semidefinite programming. Some examples are provided to illustrate the results and show some advantages of our results.

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Higher-Order Karush-Kuhn-Tucker Conditions for Firm and Weak Solutions in Nonsmooth Optimization

P. Q. Khanh¹ and <u>N. M. Tung²</u>

Abstract: This talk deals with higher-order optimality conditions of set-valued optimization problems subjects to inclusion constraints. In terms of Studniarski derivative, both higherorder necessary and sufficient optimality conditions for local firm minimizers are established without any convexity assumptions. For local weak minimizers, to analyse the critical feasible directions, we use the higher-order contingent derivatives and the corresponding asymptotic derivatives. Under some additional assumptions of directional Hölder metric subregularity and with the help of Robinson-Ursescu open mapping theorem, our multipliers are type of Karush-Kuhn-Tucker. As an application, we apply the obtained results for a nonsmooth vector optimizations with equality and inequality constraints. From there, we also get the envelope-like effect for higher-order necessary conditions. Some of our results are new and improve substantially the few results known in nonsmooth optimization.

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Carleman Estimate for Infinite Cylindrical Quantum Domains and Application to Inverse Problems

Y. Kian¹, <u>Q. S. Phan²</u>, and E. Soccorsi³

Abstract: We consider the inverse problem of determining the time independent scalar potential q of the dynamic Schrödinger equation in an infinite cylindrical domain Ω , from one Neumann boundary observation of the solution. Assuming that q is known outside some fixed compact subset of Ω , we prove that q may be Lipschitz stably retrieved by choosing the Dirichlet boundary condition of the system suitably. Since the proof is by means of a global Carleman estimate designed specifically for the Schrödinger operator acting in an unbounded cylindrical domain, the Neumann data is measured on an infinitely extended subboundary of the cylinder. Ref. [1]

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Exponents and Spectral Intervals of Dynamical Systems: Theory and Numerical Methods

V. H. $Linh^1$

Abstract: In this talk we aim to give an introduction to the theory and approximation methods for exponents and spectral intervals of continuous/discrete-time linear systems. It is known that these notions are useful tools for the stability analysis of differential equations and difference equations. First, we survey classical concepts such as Lyapunov characteristic exponents, Bohl exponents, Sacker-Sell spectrum and their properties. Then, we present numerical methods for approximating spectral intervals that are based on QR and SVD decompositions which are well known in numerical linear algebra. Finally, we discuss recent extensions of the theory and the numerical methods to differential-algebraic equations.

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A Representation of Generalized Convex Polyhedra and Applications

$\underline{\mathbf{N.~N.~Luan}}^1$ and $\mathbf{N.~D.~Yen}^2$

Abstract: It is well known that finite-dimensional polyhedral convex sets can be generated by finitely many points and finitely many directions. Representation formulas in this spirit are obtained for convex polyhedra and generalized convex polyhedra in locally convex Hausdorff topological vector spaces. Our results develop those of X. Y. Zheng (Set-Valued Anal., Vol. 17, 2009, 389–408), which were established in a Banach space setting. Applications of the representation formulas to proving solution existence theorems for infinite-dimensional linear programming problems and infinite-dimensional linear vector optimization problems are shown.

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Ba Vì, Hà Nội 21-23/4/2016

Subdifferentiation of Regularized Functions

<u>H. V. Ngai¹</u> and J-P. Penot²

Abstract: We study the Moreau regularization process for functions satisfying a general growth condition on general Banach spaces. We give differentiability criteria and we study the relationships between the subdifferentials of the function and the subdifferentials of its approximations. We also consider the Lasry-Lions process.

 $Key \ words:$ approximately convex function, convolution, monotonicity, nonsmooth analysis, regularization, subdifferential

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A Classification of Subclasses of Intuitionistic t-Norms and t-Conorms for Intuitionistic Fuzzy Sets

<u>**R. T. Ngan**¹</u>, **P. N. Chuyen**², **N. T. Manh**³, **N. T. Lam**⁴, and **B. C. Cuong**⁵

Abstract: The classification of subclasses of t-norm operators and t-conorm operators is an important result of fuzzy logics. Representable intuitionistic t-norms and representable intuitionistic t-conorms were defined and considered by Deschrijver G. et al. in [1].

In this paper, we introduce firstly a classification of subclasses of representable t-norm operators and representable t-conorm operators for intuitionistic fuzzy sets. Some properties of these subclasses are also presented.

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Continuity and Directional Differentiability of the Value Function in Parametric Quadratically Constrained Nonconvex Quadratic Programs

<u>**T. V. Nghi**¹</u> and N. N. Tam^2

Abstract: In this talk, we will present the continuity and the directional differentiability of the value function in quadratically constrained nonconvex quadratic programming problem. Our result can be used in some cases where the existing results on differential stability in nonlinear programming (applied to quadratic programming) cannot be used.

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Bán kính ổn định của hệ chuyển mạch tuyến tính tuần hoàn

L. V. $Ngoc^1$

Tóm tắt: Lý thuyết ổn định và điều khiển của các hệ động lực được nghiên cứu những năm 60 của thể kỷ 20, còn đối với hệ chuyển mạch tuyến tính, các bài toán tương tự được các nhà nghiên cứu lý thuyết và ứng dụng trên thế giới đặc biệt quan tâm khoảng thời gian 30 năm trở lại đây, việc nghiên các mô hình toán học được mô tả bởi các hệ chuyển mạch có ý nghĩa ứng dụng trong nhiều bài toán kỹ thuật và thực tiễn. Tuy nhiên, các kết quả cho hệ chuyển mạch tuyến tính vẫn còn hạn chế và nhiều bài toán vẫn chưa được khảo sát, như bài toán tính bán kính ổn định và bán kính điều khiển được của hệ chuyển mạch tuyến tính thường và suy biến chịu nhiễu, nhiễu cấu trúc.

Báo cáo trình bày tính ổn định giải tích của hệ chuyển mạch tuyến tính tuần hoàn dạng $\dot{x}(t) = A_{\sigma(t)}x(t)$ sử dụng lý thuyết Floquet và bán kính ổn định của hệ. Đóng góp của báo cáo là tính được bán kính ổn định và đưa ra ví dụ cho hệ chuyển mạch tuyến tính tuần hoàn.

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Ba Vì, Hà Nội 21-23/4/2016

An Iterative Method for Zeros of Accretive Mappings in Banach Spaces

<u>N. D. Nguyen¹</u> and N. $Buong^2$

Abstract: In this paper, for finding a zero of an accretive mapping in uniformly smooth Banach spaces, a modification of the method, introduced by L.C. Ceng, H.K. Xu and J.Ch.Yao in [1], is presented. The strong convergence of the modified method is proved under weaker conditions than those in [1]. We also show that some iterative methods in literature are special cases of our method.

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New Aspects in the Theory of Finite-Time Stability and Control of Singular Dynamical Systems

V. N. Phat¹

Abstract: Singular dynamical systems (also referred to as descriptor systems, implicit systems, differential-algebraic systems or generalized state-space systems) are a natural representation of dynamic systems and frequently appear in many practical areas such as engineering robotics, economic systems, power systems, population development systems, etc. Many mathematical control concepts have been extensively studied for many years and various interesting results have been extended to singular systems, such as Lyapunov stability and stabilization. This talk presents some new aspects in the theory of finite-time stability (non-Lyapunov stability) and control of singular dynamical systems. The report covers main concepts and contributions from the area of finite-time stability and stabilization extended to linear singular systems with time delay.

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Second-Order Optimality Conditions for Boundary Control Problems with Mixed Pointwise Constraints

N. H. Son^1

Abstract: Let Ω be a bounded domain in \mathbb{R}^N with the boundary Γ of class $C^{1,1}$ and $N \geq 2$. We consider the following semilinear elliptic optimal control problem with mixed pointwise constraints. Find a control function $u \in L^p(\Gamma)$ and a corresponding state function $y \in W^{1,r}(\Omega)$, which

$$\begin{array}{ll} \text{minimize } F(y,u) = \int_{\Omega} L(x,y(x))dx + \int_{\Gamma} \varphi(x,y(x),u(x))dx \\ (P) & \text{s.t.} \\ \begin{cases} Ay + h(x,y) = 0 & \text{in } \Omega \\ \partial_{n_A}y + \lambda y = u & \text{on } \Gamma, \\ a(x) \leq g(x,y(x)) + u(x) \leq b(x) \text{ a.e. } x \in \Gamma. \end{cases} \end{array}$$

In this talk, we present second-order optimality conditions for problem (P). We show that in case N = p = 2 and $\varphi(x, y, u) = l(x, y) + \alpha(x)u + \beta(x)u^2$ with $\beta(x) \ge \gamma > 0$ a.e. $x \in \Gamma$, there is no gap between second-order necessary optimality conditions and second-order sufficient optimality conditions.

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Radius of Approximate Controllability of Linear Retarded Systems under Structured Perturbations

N. K. Son¹, D. D. Thuan², and <u>N. T. Hong³</u>

Abstract: In this talk we shall deal with the problem of calculation of the radius of approximate controllability in the Banach state space $K^n \times L_2([-h_k, 0], K^n)$ for linear retarded systems of the form $\dot{x}(t) = A_0 x(t) + A_1 x(t-h_1) + \ldots + A_k x(t-h_k) + Bu(t)$, here $K = \mathbb{R}$ or $K = \mathbb{C}$. By using multi-valued linear operators we are able to derive computable formulas for this radius when the system's coefficient matrices are subjected to structured perturbations. Some examples are provided to illustrate the obtained results.

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Optimality Condition for $C^{1,1}$ -Vector Equilibrium Problem Based on Second-order Contingent Derivatives

T. V. Su^1

Abstract: The vector equilibrium problem provides an unified mathematical model including vector complementarity problems, vector saddle point problems, vector optimization problems and vector variational inequality problems as special cases. It plays an important role of non-linear analysis and its optimality condition is an important subject to study. Up still now, results in second-order conditions of vector equilibrium problems have taken a main part in the achievements for optimality conditions.

In this talk, we give some necessary and sufficient optimality conditions for local weakly efficient solution of vector equilibrium problem with constraints (shortly, VEPC) defined by a set constraint, a generalized inequality constraint and an equality constraint by means of second-order contingent derivatives. First of all, we introduce the definitions of second-order contingent derivative and second-order asymptotic contingent derivative. Next we introduce the necessary optimality conditions for VEPC and an example to illustrate for obtained results. Finally, the sufficient optimality conditions for VEPC are also derived.

Notice that the constraints functions will be used here of $C^{1,1}$ and the equality constraint function is directionally metrically regular.

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On the Solution Existence of Nonconvex Quadratic Programming Problems in Hilbert Spaces

$\underline{\mathbf{N.~N.~Tam}}^1$ and $\mathbf{V.~V.~Dong}^2$

Abstract: In this talk, we consider the quadratic programming problems under finitely many convex quadratic constraints in Hilbert spaces. By using the Legendre property of quadratic forms or the compactness of operators in the presentations of quadratic forms, we establish some sufficient conditions for the solution existence of the considered problems. As special cases, we obtain some existence solution results for the quadratic programming problems under linear constraints in Hilbert spaces.

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Monotonic Optimization for the Knapsack Problem

<u>**T. V. Thang**¹ and **H. Tuy**²</u>

Abstract: We consider the Knapsack problem in the form of the following nonlinear integer programming problem (MNKP):

$$\max \quad f(x) = \sum_{j=1}^{n} f_j(x_j)$$

s.t.
$$g_i(x) = \sum_{j=1}^{n} g_{ij}(x_j) \le w_i, \quad i = 1, 2, ..., m,$$
$$x \in X = \{x \in \mathbb{Z}^n | a_j \le x_j \le b_j, \ j = 1, 2, ..., n\}$$

where \mathbb{Z}^n is the set of integer points in \mathbb{R}^n , $a_j, b_j \in \mathbb{Z}$, $a_j \leq b_j$ for any $j = 1, 2, ..., n, f_j$: $[a_j, b_j] \to \mathbb{R}, j = 1, ..., n$ are inreasing functions and $g_{ij} : [a_j, b_j] \to \mathbb{R}, i = 1, 2, ..., m, j = 1, 2, ..., n$, are inreasing lower semicontinuous functions.

Knapsack problems have been widely studied because of numerous applications in various fields, such as capital budgeting, marketing, production planning. In this paper we propose to solve (MNKP) by applying the branch-reduce-and-bound algorithm for general discrete monotonic optimization as was developed by H. Tuy. Exploiting the fact that all the functions involved are separable, the Lagrangian relaxation method is used to compute an upper bound for the subproblem generated in each iteration. Also, since (MNKP) is a integer programming, the separation and reduction cuts are easily computed. Finally, to speed up the convergence an adaptive subdivision process is used for partionning instead of the standard bisection. Comparison results with other existing method are also reported.

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Building some Algorithms and Programs on Maple to Predict and Correct the Numerical Solutions of Cauchy Problems of Ordinary Differential Equations

<u>N. H. Thanh¹</u> and T. T. Tu^2

Abstract: We know that finding analytical solutions of ordinary differential equations is very difficult. Even some equations in the Riccati's class cannot be solved analytically. Therefore, evaluating the roots of these equations numerically has been popular in science and technology nowadays.

Many researches on numerical integration of Cauchy problem for differential equations have been studied since the 1960s. The theory of numerical methods for the initial value problems has been investigated by so many researchers such as J. D. Lambert, J. C. Butcher, G. Dahlquist, etc. For a long time, it has been one of the most popular fields of differential equations. Now, whenever discussing about numerical methods, we can refer to a lot of well-known monographs and papers such as [1], [2], [3]. In this note, the theory of numerical methods for differential equations is used as a tool to set up some algorithms for finding the numerical solutions of Cauchy problems. Based on these algorithms, we construct programs on Maple that give numerical solutions to a class of Cauchy problems. In addition, the comparisons between the roots found by our algorithms and by using Maple are also considered. The default method on Maple for initial value problems (see [4]) is a Runge-Kutta-Fehlberg method that produces a fifth order accurate solution. Most of the results are illustrated graphically.

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Boundedness in Weighted L_p Spaces for the Kontorovich-Lebedev-Fourier Generalized Convolution Operators and Applications

N. X. Thao¹, T. Tuan², and <u>P. V. Hoang³</u>

Abstract: In this paper, several weighted L_p -norm inequalities (p > 1) for the Kontorovich-Lebedev generalized convolutions are obtained. With the help of these inequalities, we will consider a partial differential equation of second order of parabolic type with unbounded coefficients in unbounded domains without uniform ellipticity condition, and study the boundedness of its solution in weighted L_p spaces (p > 1). Moreover, the boundedness of a scattered acoustic field in weighted L_p spaces (p > 1) is obtained by basing on these new inequalities.

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Hội thảo Tối ưu và Tính toán Khoa học lần thứ 14

An Optimization Problem of Fractional Signal Processing

<u>**T.** H. Thao¹</u> and H. T. P. Thao²

Abstract: The aim of this paper is to find an optimal state estimation for a fractional signal, where the noise caused by a fractional Brownian motion. An approximation approach is introduced in order to lead the problem into that of can be considered under an usual context of filtering theory, where the signal is perturbed by a Gaussian white noise. The solution of the signal equation is found and a filtering equation for fractional signal is established as well.

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Online Identification of Continuous Piecewise Affine Dynamical Systems

<u>L. Q. Thuan¹</u>, T. V. D. Boom², and S. Baldi³

Abstract: System identification has been a long-standing problem in systems and control theory and received much attentions. The field of classical system identification uses statistical methods to build mathematical models of the dynamical systems from measured data. For piecewise affine (PWA) systems, system identification is composed of two ingredients: the estimation of subsystem parameters and of the hyperplanes defining the partition. In the case that one of the ingredients is assumed to be known, various contributions have been presented in the literature. Identifying piecewise affine systems with known partitions can be carried out by standard linear identification techniques in a local manner. When both subsystems and the partition are unknown, to identify PWA systems, the partition must be estimated together with the subsystems. This has been known a very challenging problem and the main difficulty lies in the fact that the identification problem includes a classification problem to determine in which region each data point must be associated. This task must be done in the condition of the lack of information and the machine learning technique is employed with slowly convergent rate. Despite of the difficulty, there are recently proposed techniques dealing with the issue: Bayesian procedure, the bounded-error procedure, the clustering-based procedure and the Mixed-Integer Programming procedure. Most of the work in the area of identifying PWA systems focuses on the development of identification algorithms for discrete-time piecewise affine functions in regression form and the algorithms are offline.

In the talk, we will present our recent results on the online identification of continuous piecewise affine dynamical systems in the state space forms with jointly unknown partition and subsystem matrices. The partition of the system is generated by the so-called centers. By representing continuous piecewise affine systems in the so-called max-form and using the recursive Gauss-Newton algorithm for a suitable cost function, we derive adaptive laws to online estimate parameters including both subsystem matrices and centers. The effectiveness of the proposed approach is demonstrated with some numerical examples.

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Ảnh hưởng của cạnh tranh trực tiếp và gián tiếp lên hệ sinh thái quần thể có cấu trúc

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Tóm tắt: Trong báo cáo này, chúng tôi trình bày một mô hình hệ sinh thái hai loài thú (loài thú 1 và loài thú 2) và một loài mồi trong đó các quần thể thú được chia làm hai loại: thú non và thú trưởng thành. Loài thú 1 và loài thú 2 cạnh tranh gián tiếp với nhau khi sử dụng loài mồi như một nguồn thức ăn chung. Hai loài thú này cũng cạnh tranh trực tiếp với nhau theo nghĩa loài thú 1 trưởng thành có thể ăn con non của loài thú 2 và ngược lại. Mô hình mô tả hệ sinh thái này là một hệ gồm 5 phương trình vi phân thường. Chúng tôi nghiên cứu sự tồn tại, tính ổn định địa phương và toàn cục của điểm cân bằng của mô hình trong miền không âm. Chúng tôi cũng thảo luận ý nghĩa sinh thái của các kết quả đạt được.

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Exponential Stability of Functional Differential Systems^{*}

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Abstract: We present a novel approach to exponential stability of functional differential systems. Our approach is relied upon the theory of positive linear functional differential systems and a comparison principle. Consequently, we get some comparison tests and explicit criteria for exponential stability of functional differential systems. Two examples are given to illustrate the obtained results.

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Recent Developments of Global Optimization

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Abstract: During the last fifteen years, tremendous progress has been achieved in Global Optimization, totally transforming the landscape of this field. The talk is about the three following major recent theoretical and computational developments:

- 1) Problems with hard nonconvex constraints;
- 2) Monotonic Optimization;
- 3) Fixed point, minimax and equilibrium problems.

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