DANH SÁCH BÁO CÁO

TÓM TẮT BÁO CÁO

Phương pháp số giải bài toán elliptic với điều kiện biên hỗn hợp mạnh trong nửa dải

Đặng Quang Á[1], Trần Đình Hùng[2]

Các bài toán biên trong miền vô hạn (không giới nội) xuất hiện nhiều trong các lĩnh vực vật lý, cơ học, môi trường,... Để giải số các bài toán này người ta thường giới hạn bài toán trong một miền hữu hạn và tìm cách thiết lập các điều kiện biên trên biên nhân tạo hoặc sử dụng lưới tính toán tựa đều ánh xạ miền không giới nội vào miền giới nội. Khác với các cách làm trên, chúng tôi tiếp cận tới bài toán trong miền không giới nội bởi hệ vô hạn các phương trình đại số tuyến tính. Trong hướng này chúng tôi đã thu được một số kết quả đối với các bài toán một chiều. Mới đây chúng tôi cũng đã thành công trong việc giải bài toán elliptic trong nửa dải nhờ sử dụng ý tưởng của của Polozhii trong phương pháp biểu diễn tổng để đưa hệ vô hạn các phương trình véc tơ về các hệ vô hạn các phương trình vô hướng.

Phát triển các ý tưởng trên với phương pháp chia miền, trong báo cáo này chúng tôi xây dựng phương pháp giải bài toán biên hỗn hợp mạnh, trong đó có điểm thay đổi loại điều kiện trên một biên vô hạn cho phương trình elliptic trong nửa dải. Một số thực nghiệm tính toán đã chứng tỏ tính hữu hiệu của phương pháp được đề xuất.

Từ khóa: miền vô hạn, phương pháp chia miền, hệ vô hạn, bài toán biên hỗn hợp.

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DC Programming and DCA: Theory, Algorithms and Applications

Le Thi Hoai An[1]

DC (Difference of Convex functions) Programming and DCA (DC Algorithms), which constitute the backbone of Nonconvex Programming and Global Optimization, were introduced by Pham Dinh Tao in their preliminary form in 1985 and extensively developed by Le Thi Hoai An and Pham Dinh Tao since 1994 to become now classic and increasingly popular (see for example http://lita.sciences.univ-metz.fr/~lethi/). Their popularity resides in their robustness, inexpensiveness, flexibilty, versatility and performance compared to existing methods, their adaptation to specific structures of addressed problems and their ability to solve real-world large-scale nonconvex programs.

DC Programming and DCA address the problem of minimizing a function f which is the difference of two convex functions on the whole finite dimensional Euclidean space X or on a convex set C contained in X. If $f = g - h$ then $g - h$ is a DC decomposition of f and g, h DC components of f. DC Programming is a natural and logical extension of Convex Programming, sufficiently large to cover almost all nonconvex programs but not too in order to use the powerful arsenal of modern Convex Analysis and Convex Optimization. The set of DC functions on X, denoted DC(X), is the smallest vector space containing the "convex cone" C(X) of lower semicontinuous proper convex functions on X. DC(X) is large enough to contain most real-life objective functions and is stable with respect to usual operations in optimization.

DC programming investigates the structure of DC(X), DC duality and local and global optimality conditions for DC programs. A dual DC program, defined with the help of the conjugate functions g^*, h^* of g and h respectively, is the minimization of $h^* - g^*$ on X. Relations between local/global solutions of primal and dual DC programs are quite simple. The complexity of DC programs clearly lies in the distinction between local and global solution and, consequently, the lack of verifiable global optimality conditions.

DCA is based on DC duality and local optimality conditions. The philosophy of DCA is to bring back DC(X) to C(X): solving a DC program by as sequence of approximate convex subprograms.
DCA's distinctive feature relies upon the fact that DCA deals with the convex DC components g and h but not with the DC function f itself. Moreover, a DC function f has infinitely many DC decompositions which have crucial implications for the qualities (speed of convergence, robustness, efficiency, globality of computed solutions,...) of DCA.
DCA is a descent method without linesearch (so very appreciated in large-scale problems) but with global convergence. Remark that with appropriate DC decompositions, *DCA permit to recover, as special cases, most standard algorithms in Convex/Noncovex Programming*.
These theoretical and algorithmic tools have been successfully used by many researchers and practitioners to model and solve their nonconvex programs in different fields of Applied Sciences.
Extension of DCA to DC programs with DC constraints by using penalty techniques/ linearizing DC constraints (as in DCA for standard DC programs) with updating penalty parameter, exact penalty and error bounds in DC programming, DC relaxation for lower bounding in Branch-and-Bound techniques applied to DC programming, have been constantly investigated during the last decade.

This talk on DC programming and DCA will be comprised of four parts. First we outline basic theoretical and algorithmic tools for an easy understanding of the DC Programming and DCA's philosophy, while in the second part we show how to solve several classes of hard problems in Combinatorial Optimization and Operations Research by DC Programming and DCA. The third one is devoted to applications of these theoretical and algorithmic tools for modelling and solving some real world nonconvex programs in Machine Learning, Image processing, Communication Systems, Finance, Transport-Logistic and Supply Chain Management. Finally we discuss about some open issues in DC programming and DCA.

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On the Hölder Continuity of Approximate Solutions Mappings to Parametric Vector Equilibrium Problems

Lâm Quốc Anh[1]

We establish verifiable sufficient conditions for Hölder continuity of approximate solutions to parametric equilibrium problems, when solutions may be not unique. The results are shown to be extensions of recent ones for equilibrium problems with some improvements. As applications, we derive this Hölder continuity for constrained minimization, variational inequalities and fixed point problems.

Keywords: Hölder continuity, Approximate solutions, Vector equilibrium problems, Optimization problems, Variational inequalities.

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Semicontinuity of Solution Sets to Parametric Vector Equilibrium Problems Involving the Lorentz Cone

Lam Quoc Anh[1], Nguyen Huu Danh[2]

We consider vector equilibrium problems involving the Lorentz cone. Sufficient conditions for the solution sets to be upper semicontinuity are established. Applications to some important problems are also provided.

Keywords: Lorentz cone, Upper semicontinuity, Equilibrium problems, Optimization problems, Variational inequalities.

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Parallel Methods for Solving systems of Equations in Banach Spaces

Pham Ky Anh[1], Nguyen Buong[2], and Dang Van Hieu[3]

Various problems of science and engineering, including multi-parameter identification problems, convex feasibility problems, common fixed point problems, etc..., lead to systems of ill-posed operator equations. Very recently, several sequential and parallel regularizing methods for solving such systems of operator equations in Hilber spaces have been proposed.

In this report we investigatesome parallel iterative regularization methods for solving systems of equations involving *m*-accretive operators in Banach spaces. The report is based on our paper [1] and extended previous results announced in [2-4].

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Parallel and Sequential Hybrid Methods for a Finite Family of Quasi ϕ-Asymptotically Nonexpansive Mappings

Pham Ky Anh[1], Dang Van Hieu[1]

In this talk, we propose some novel parallel and sequential hybrid methods for finding a common fixed point of a finite family of quasi ϕ-asymptotically nonexpansive mappings.The idea of these methods is to combine Mann's (or Halpern's) iterative algorithm, parallel splitting-up technique, monotone hybrid iteration method, and projection-iteration technique. The results presented here modify and extend some previous results obtained by several authors.

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Strong Convergence Theorem for an Equilibrium Problem and a Nonexpansive Semigroup in Hilbert Spaces

Nguyen Buong[1], Nguyen Dinh Duong[2]

Let C be a nonempty closed and convex subset of a real Hilbert space H and let $S = \{T(t):\ 0 \le t < \infty\}$ be a nonexpansive semigroup on C. We denote by $F(S)$ the set of all common fixed points of S.

The equilibrium problem of $G: C \times C \to \mathbb{R}$ is to find $x^* \in C$ such that $G(x^*; y) \ge 0$ for all $y \in C$.

The set of such solution is denoted by $EP(G)$. For solving equilibrium problem, we assume that the bifunction G satisfies the following conditions:

(A1) $G(x;\ x) = 0$ for all $x \in C$;

(A2) G is monotone, i.e., $G(x;\ y) + G(y;\ x) \le 0$ for any $x, y \in C$;

(A3) lim $\limsup_{t\to 0} G((1-t)x + tz, y) \le G(x, y)$, for each $x, y, z \in C$;

(A4) For each $x \in C$, $G(x;.): C \to \mathbb{R}$ is convex and lower semicontinuous.

In this report, we introduce a new iterative method for finding $p \in F(S) \cap EP(G)$, and prove the strong convergence theorem in Hilbert spaces.

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Regularization for a Generalized Complementarity Problems

Nguyen Buong[1], Nguyen Thi Thuy Hoa[2]

In this report, we use the Tikhonov regularization method to solve a constrained generalized complementarity problem that is to find an element

$$\tilde{x} \in C: \; g(\tilde{x}) \le 0, \; h(\tilde{x}) \le 0, \; \langle g(\tilde{x}), h(\tilde{x}) \rangle_{E^m} = 0,$$

where C is a convex closed subset in E^m, $g(x)$ and $h(x)$ are two continuous funtions from an Euclidian space E^m to E^m and $\langle . \, , . \rangle_{E^m}$ denotes the scalar product of E^m.

Convergence and convergence rates of regularized solutions are established under ordinary condition in the theory of nonlinear ill-posed problems. A numerical example is also give for illustration.

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Regularization Methods for Variational Inequalities over the Set of Common Fixed Points of Nonexpansive Semingroups on Banach Spaces

Nguyen Buong[1], Nguyen Thi Thu Thuy[2], <u>Pham Thanh Hieu</u>[3]

Let E be a uniformly convex and uniformly smooth Banach space. Denote by E^* and $\langle x; x^* \rangle$ respectively, the dual space of E and the value of $x^* \in E$ at $x \in E$. In this report, we introduce a regularization scheme based on Browder-Tikhonov regularization method and combine the proposed regularization method with iterative process and inertial proximal point algorithm for solving the following class of variational inequalities: find $p_* \in \wp$ such that

$$\langle F(p_*), J(p_* - p) \rangle \le 0 \quad \forall p \in \wp;$$

where $\wp$ is the set of common fixed points of a nonexpansive semigroup $\{T(s): s > 0\}$ on E, $F: E \to E$ is a nonlinear mapping, and $J: E \to E_*$ is the normalized duality mapping of E. The first method is Browder-Tikhonov regularization: find $x_n \in E$ such that

$$A_n(x_n) + \varepsilon_n F(x_n) = 0, \tag{1}$$

where $A_n = I - T_n$, $T_n x = \frac{1}{t_n} \int_0^{t_n} T(s) x dx, x \in E$, and $\{t_n\}, \{\varepsilon_n\}$ are two sequences of positive real numbers such that $t_n \to \infty$ and $\varepsilon_n \to 0$ as $n \to \infty$.

The second one is a combination of explicit iterative scheme and regularization method (1)

$$w_{n+1} = w_n - \beta_n \left[A_n(w_n) + \varepsilon_n F(w_n) \right], n \ge 1; w_1 \in E,$$

where β_n satisfies some conditions.

In the third one, we combine the inertial proximal point method with the regularization scheme (1), as follows:

$$c_n \left[A_n + \varepsilon_n F \right](z_n + 1) + z_{n+1} - z_n = \gamma_n (z_n - z_{n-1}),\ z_0;\ z_1 \in E,$$

where $\{c_n\}$ and $\{\gamma_n\}$ are two sequences of positive numbers.

Strong convergence theorems are proved, based on the condition that F is a strongly accretive and strictly pseudocontractive mapping and other conditions posed on the sequences of parameters.

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Newton-Kantorovich Regularization for a Finite System of Nonlinear Ill-Posed Equations Involving Accretive Mappings

Nguyen Buong[1], Nguyen Thi Thu Thuy[2], Nguyen Duong Nguyen[3]

In this report, we propose a new regularization methods of Newton-Kantorovich iterative type for a finite system of nonlinear ill-posed operator equations involving accretive and Lipschitz continuous mappings on real, reexive and strictly convex Banach space E with a uniformly Gâteaux diferentiable norm.

The strong convergence of the methods is proved without the Lipschitz continuous condition on the derivatives of the mappings and the weak sequential continuous property for the duality mapping of E.

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Về tính chất của nửa nhóm bị nhiễu và một vài ứng dụng

Đặng Đình Châu[1]

Trong không gian Banach X chúng ta xét các họ toán tử tuyến tính liên tục mạnh $U(t,s)_{t\geq s}$. Chúng tôi xin nhắc lại rằng họ toán tử tiến hóa của bài toán Cauchy đặt chỉnh đều có các tính chất sau:

1. $U(t,t)=I$ với mọi $t\geq 0$;
2. $U(t,s).U(s,\tau)=U(t,\tau)$ với mọi $t\geq s\geq \tau\geq 0$;
3. Ánh xạ $(t,s)\to U(t,s)x$ là liên tục mạnh đối với mỗi $x\in X$;
4. $\|U(t,\ s)\|\leq Ne^{\omega(t-s)}$ với các hằng số dương N,ω độc lập với $t\geq s\geq 0$.

Chúng ta sẽ giả thiết họ các toán tử tiến hóa $U(t,\ s)_{t\geq s}$ được sinh ra từ phương trình tiến hóa đặt chỉnh sau đây:

$$\frac{du(t)}{dt}+A(t)u(t)=0 \text{ với } t\geq s\geq 0;\quad u(s)=x_s \text{ với } x_s\in X.$$

Trong báo cáo này chúng tôi đã sử dụng các phương pháp nửa nhóm liên tục mạnh và các phương pháp thông dụng của lý thuyết định tính phương trình vi phân để nghiên cứu dáng điệu tiệm cận nghiệm của các họ toán tử tiến hóa bị nhiễu và ứng dụng vào mô hình dân số phụ thuộc vào tuổi dạng:

$$\frac{\partial f}{\partial t}(a,t)+\frac{\partial f}{\partial a}(a,\ t)+\mu(a)f(a,\ t)=0 \text{ với } a,t\geq 0;$$

$$f(0,t)=\int_0^\infty \beta(a)f(0,\ t)da \text{ với } t\geq 0;$$

$$f(a,0)=f_0(a) \text{ với } a\geq 0.$$

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Convexity of Sets and Functions via Second-Order Subdifferentials

Nguyen Huy Chieu, J.-C. Yao, Nguyen Dong Yen[1]

It is proved that, for a set belonging to certain classes of closed sets in Asplund spaces, the positive semidefiniteness of the limiting second-order subdifferential of its indicator function at each boundary point is necessary and sufficient for the local convexity of the set. It is also shown that the positive semidefiniteness of the limiting second-order subdifferentials of some kinds of continuous functions can characterize their convexity on closed convex sets.

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Metric Regularity of a Positive-Order for Implicit Multifunctions

Thai Doan Chương[1]

In this talk, we first provide new sufficient conditions for an implicit multifunction to achieve the metric regularity (resp., metric sub- regularity) of a positive-order. Then we show that such sufficient condi-tions turn out to be also necessary for the metric regularity (resp., metric subregularity) of a positive-order of the implicit multifunction when the corresponding parametric multifunction is (locally) convex and closed.

Key Words: Implicit multifunction, metric q-regularity, coderivative

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Phương pháp lặp song song dạng Runge-Kutta-Nystrom hai bước một dựa trên các điểm trùng khớp Gauss-Legendre

Nguyễn Hữu Công[1], La Trí Dũng[2]

Báo cáo trình bày về một lớp phương pháp dự báo-hiệu chỉnh song song để giải bài toán giá trị đầu không cương của các hệ phương trình vi phân thường cấp hai dạng đặc biệt $y'' = f(t, y)$. Phương pháp này được đặt tên là phương pháp lặp song song dạng Runge-Kutta-Nystrom hai bước một dựa trên các điểm trùng khớp Gauss-Legendre (TBTPIRKNG - two-step-by-two-step parallel iterated Runge-Kutta-Nystrom methods based on Gauss-Legendre collocation points).
Phương pháp dự báo dựa trên các công thức Adam.
Phương pháp hiệu chỉnh TBTRKNG được xây dựng từ các hệ số của hai phương pháp Runge-Kutta-Nystrom: một phương pháp RKN s nấc dựa trên véc tơ trùng khớp Gauss-Legendre $\hat{c} = \left(\hat{c}_1, \hat{c}_2, ..., \hat{c}_s\right)$, và một phương pháp RKN trùng khớp 2s nấc dựa trên véc tơ trùng khớp $c = \left(\hat{c}_1, \hat{c}_2, ..., \hat{c}_s, \hat{c}_1 + 1, \hat{c}_2 + 1, ..., \hat{c}_s + 1\right)$. Tại bước lặp thứ n, các giá trị nấc của phương pháp RKN trùng khớp 2s nấc tính tại $t_n + (\hat{c}_1 + 1)h, t_n + (\hat{c}_2 + 1)h, ..., t_n + (\hat{c}_s + 1)h$ được sử dụng làm véc tơ nấc cho phương pháp RKN s nấc tại bước $n + 2$. Bằng cách đó ta thu được phương pháp hiệu chỉnh có thể tính hai bước một (two-step-by-two-step) trong một lần lặp.
Báo cáo sẽ đề cập tới cấp chính xác, tính ổn định, sự hội tụ của phương pháp hiệu chỉnh TBTRKNG cũng như của bản thân phương pháp TBTPIRKNG.
Hai phương pháp TBTPIRKNG (với s = 3 và s = 4) với bước lưới cố định sẽ được áp dụng vào ba bài toán thử được sử dụng rộng rãi là NEWT, LINE và FEHL. Các thực nghiệm số đã chứng tỏ phương pháp TBTPIRKNG tỏ ra hiệu quả hơn nhiều so với các phương pháp lặp song song RKN và các codes ODEX2nvà DOP853đã biết.

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Parallel-Iterated Pseudo Two-Step Runge Kutta Methods with Step Size Control

Nguyen Huu Cong[1] and Nguyen Thu Thuy[2]

The aim of this paper is to develop a class of constant step size parallel-iterated pseudo two-step RK methods (PIPTRK mehtods) for nonstiff first-order ODE problems into variable step size methods. Embedded formulas are provided for giving a cheap error estimate used in the step size control. Methods with variable parameters approach were applied for overcoming the diffculty in using two-step methods with variable step size. By applications to a few widely used test problems, we compare the efficiency of the resulting PIPTRK methods with step size control (PIPTRKSC methods) with the codes PIRK, DOPRI5, DOP853 and ODEX. This numerical comparison shows that these new PIPTRKSC methods are by far superior to the PIRK, DOPRI5, DOP853 and ODEX codes.

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Giải thuật tiến hóa và ứng dụng trong việc xấp xỉ lời giải của bài toán tối ưu đa trị

Vũ Chí Cường[1], Bùi Thu Lâm[2]

Tối ưu hóa là một trong những lĩnh vực nghiên cứu kinh điển của toán học có ảnh hưởng đến hầu hết các lĩnh vực khoa học-công nghệ và kinh tế-xã hội. Việc nghiên cứu những vấn đề liên quan đến tối ưu hóa đã và đang là sự quan tâm của nhiều nhà khoa học trong và ngoài nước. Đồng thời việc tìm ra các giải pháp cho bài toán tối ưu cũng trở thành một chủ đề quan trọng trong lĩnh vực tính toán.

Có rất nhiều các phương pháp tính toán được đề xuất để tìm giải pháp cho các bài toán tối ưu hóa trên máy tính. Đặc biệt với các bài toán khó, xu hướng ứng dụng các kỹ thuật của trí tuệ nhân tạo đã được triển khai rộng rãi. Một trong số đó là kỹ thuật tính toán tiến hóa (Evolutionary Algorithms – EAs).

Trong báo cáo này, chúng tôi đề xuất một thuật toán tiến hóa sử dụng thông tin định hướng mà chúng tôi gọi là DEAL (Direction-guided Evolutionary ALgorithm) và ứng dụng nó trong việc giải quyết các bài toán tối ưu đa trị (multimodal optimization problems). DEAL được xây dựng dựa trên những tính toán khoa học nhằm tổ chức, quản lý và khai thác tập các giả pháp ưu tú (elite set). Tại mỗi thế hự, các giải pháp khả thi được lựa chọn để tiến hóa theo hai hướng: 1) Hướng hội tụ: là hướng từ một giải pháp thông thường đến một giải pháp ưu tú và 2) Hướng phân tán: là hướng giữa hai giải pháp ưu tú. Các kết quả thực nghiệm của thuật toán trên lớp 20 bài toán tối ưu đã trị của "Benchmark Functions for CEC'2013 Special Sesion and Competition on Niching Method for Multimodal Function Optimization" đã cho thấy những khả quan của hướng nghiên cứu này.

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Error Bounds for Nonsmooth Definable Functions in O-Minimal Structures

Hoang Phi Dung[1]

In this report we introduce some generalizations of Farkas lemma in sequential forms. These are established without any constraint qualification conditions and are proved to be equivalent to some new approximate Hahn-Banach theorems. The results can be used to derive optimality conditions and duality results for optimization problems in the absence of constraint qualification conditions.

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On the Asymptotic Behavior of the Model of Age-Dependent Population

Nguyen Canh Duy[1], Dang Dinh Chau[2]

Suppose X a Banach space, the following evolution equation:

$$\frac{dx(t)}{dt} = A_1(t)x(t) \text{ for } t \geq 0; \tag{1}$$

$$\frac{dy(t)}{dt} = A_2(t)y(t) \text{ for } t \geq 0. \tag{2}$$

here $x(.), y(.) \in X$, $A_1(t)$ and $A_2(t)$ $(t \in \mathbb{R}^+)$ is the linear operator from X to X. We assume that $A_1(t)$ and $A_2(t)$ satisfy the conditions for the evolution equations (1) and (2) adjustment is set. The first result of this problem is given by N. Levinson in 1946 . The content of this article is to present an extension of Levinson's theorem for evolution equations in Banach spaces. Consider the differential equations:

$$u'(t) = A(t)u(t), \tag{3}$$

where $A(t)$ is closed, generally unbounded linear operators on a Banach space X. Let Eq. (3) be well-posed, that is, it generates an evolutionary operator $U(t;\ s)_{t \geq s}$ consisting of bounded linear operators from X to itself with the following properties:

i) $U(t,t) = I \quad \forall t \geq 0$; $\|U(t,\ s)\| \leq Ne^{\omega(t-s)} \ \forall t \geq s$;

ii) $U(t,r)U(r,s) = U(t,s) \quad (t \geq r \geq s)$;

iii) $U(t,s)$ is continuous in $(t;s)$ for every fixed x $(t \geq s)$.

We consider integral equations of the form

$$x(t) = U(t,s)x(s) + \int_s^t U(t,\tau)B(\tau)x(\tau)d\tau \quad \forall t \geq s,$$

where $B(.):[0;\infty) \to L(X)$ is strongly continuous and satisfies $\int_0^\infty \|B(\tau)\| d\tau < +\infty$.

In this report we study the stability of the solutions of Eq. (2). The obtained results will be used to study the asymptotic behavior of the age-dependent population with spatial diffusion.

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Stabilities of Deterministic and Stochastic Dynamical Equations on Time Scales

Nguyen Huu Du[1]

In 1988, the theory of dynamic equations on time scales was introduced by Stefan Hilger in order to unify continuous and discrete calculus. Since then, there have been many papers investigating on analysis and dynamic equations on time scales, not only to **unify** trivial cases, that is ODEs and OΔEs, but also to **generalize** these results.

Furthermore, in view of numerical analysis, a times scale can be considered as a set of points where we want to estimate the values of solutions to a dynamical system.

In this talk, we survey some main points of analysis on time scales. We also consider some approximative problems for (random) dynamical equations on time scales.

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Optimizing the Bus Network Configuration in Da Nang City

Nguyen Ngoc Dai[1], Nguyen Thanh Hao[2], Nguyen Quang Thuan[1], Vu Thi Huong[2]

We introduce mixed Pareto quasi – variational inclusion problems and show some sufficient Bus network system, an indispensable component of urban transportation, plays a key role in solving the situation of traffic congestion as well improving the quality of transport service to any city. This work adapted the model proposed by Hiroshi Shimamoto et al. [1] in order to optimize the bus network configuration and its frequencies in Da Nang city. This model is formulated as a bi-level problem in which the lower problem is considered with the common lines problem while the upper lever is a multi-objective optimization to minimize two main objectives: cost by passengers and cost of bus service operator. A shortest hyperpath procedure is used to solve traffic assignment in the lower level. For the upper level, we use the genetic algorithm NSGA-II to find the Pareto front. We test the transportation network in Da Nang city with numerous random traffic demand instances. The results show that our solutions surpass what the Da Nang authorities proposed.

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Distributed Convex Optimization Using Subgradient and Accelerated Proximal Gradient Methods

Minh Dang Doan[1]

In this paper, we provide an algorithm for solving a class of large-scale convex optimization problems, together with the convergence analysis. The algorithm is suitable for distributed implementation, and we can define a priori number of iterations to adjust the bounds on constraint violation and suboptimality. We consider the following problem:

$$f^* = \min_x f(x) \tag{1}$$

$$\text{s. t. } g(x) \le 0, \tag{2}$$

$$x \in X, \tag{3}$$

in which:

- $f : \mathbb{R}_n \to \mathbb{R}$ is a convex function.
- $g = \left[g_1 \ldots g_m\right]^T$, with $g_i : \mathbb{R}^n \to \mathbb{R}$ convex for all $i \in \{1,\ldots,m\}$.

The constraint set defined by (2) is assumed to have nonempty interior, and we have a vector $\tilde{x}$ such that the Slater condition holds, i.e. $g_i\left(\tilde{x}\right) < 0, \quad \forall i$.

- $X \in \mathbb{R}^n$ is a hyperbox.

The couplings in the constraint functions $g_i\left(x\right)$ often yield trouble for distributed optimization algorithms, hence we will treat the dual problem of (1)–(3). Firstly, we form the Lagrangian function to relax the coupled constraints

$$L(x,\mu) = f(x) + \mu^T g(x), \quad \mu \in \mathbb{R}_+^m$$

and assume that $\nabla_x L$ is Lipschitz continous, i.e. $\exists\, L$ such that

$$\left\|\nabla_x(x_1,.) - \nabla_x(x_2,.)\right\|_2 \le L\left\|x_1 - x_2\right\|_2, \forall x_1, x_2 \in X.$$

Now we aim to solve the following dual problem

$$q^* = \max_{\mu \in \mathbb{R}_+^m} q(\mu), \text{ with } q(\mu) = \min_{x \in X} L(x,\mu)$$

using an approximate subgradient method for maximizing the dual function, and an accelerated proximal gradient method for minimizing the Lagrangian in each dual iteration.

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Sequential Farkas Lemmas and Applications

Nguyen Dinh[1], **Tran Hong Mo**[2]

In this report we introduce some generalizations of Farkas lemma in sequential forms. These are established without any constraint qualification conditions and are proved to be equivalent to some new approximate Hahn-Banach theorems. The results can be used to derive optimality conditions and duality results for optimization problems in the absence of constraint qualification conditions.

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Farkas Lemma for Convex Systems Revisited and Applications to Sublinear-Convex Optimization Problems

Nguyen Dinh[1], Tran Hong Mo[2]

In this report we establish new versions of Farkas lemma for systems which are convex with respect to a cone and convex with respect to an extended sublinear function under some Slater-type constraint qualification conditions and in the absence of lower semi-continuity and closeness assumptions on the functions and constrained sets. The results can be considered as counterparts of some of the earlier corresponding results in [1]. As consequences, we get extensions of Hahn-Banach theorem for extended sublinear functions (the situation where the celebrated Hahn-Banach theorem failed). The results obtained are then applied to obtain duality results and optimization conditions for a class of composite problems involving sublinear-convex mappings.
Two special cases are examined at the end of the paper. In the _rst one our results give rise to some generalized Fenchel duality theorems while in the second one, in normed spaces, our result leads to the separation theorem for convex sets (not necessarily closed) in normed spaces.

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Phương pháp biến đổi thích nghi giải một số bài toán rời rạc

Vũ Thanh Hà[1]

Nói chung, các bài toán tối ưu rời rạc hay các bài toán quy hoạch nguyên thường được giải bằng phương pháp nhánh cận và sử dụng thuật toán quay lui để duyệt và loại bỏ các tập con chắc chắn không chứa nghiệm tối ưu. Phương pháp nhánh cận đã là sự cải tiến đáng kể so với phương pháp đơn giản là duyệt toàn bộ. Đặc điểm của các bài toán tối ưu rời rạc là các thành phần của phương án đều là số nguyên, nên rất khó áp dụng các phương pháp liên tục để giải. Tuy nhiên, trong một số trường hợp cụ thể, ta vẫn có thể sử dụng phương pháp liên tục để giải một số bài toán rời rạc. Phương pháp này, nói chung phức tạp về mặt thuật toán, nhưng lại cho thời gian tính nhanh hơn.

Báo cáo này đề cập đến phương pháp sử dụng thuật toán biến đổi thích nghi của các Giáo sư R. Gabasov và F. M Kirillova để giải một bài toán rời rạc - bài toán "cái túi", là bài toán tối ưu rời rạc có dạng:

$$\begin{cases} f(x) = \sum_{j=1}^{n} c_j x_j \to \max \\ \sum_{j=1}^{n} a_j x_j \le b, 0 \le x_j \le d_j^*, x_j \in Z^+, j = \{1,2,...n\} \end{cases}$$

Phương pháp được đề xuất tìm được lời giải tối ưu nguyên toàn cục mà không cần sử dụng thuật toán "nhánh và cận". Độ phức tạp của thuật toán là đa thức.

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Identification of Nonlinear Heat Transfer Laws from Boundary Observations

Đinh Nho Hao[1], Bui Viet Huong[2], D. Lesnic

There are many physical phenomena occurring at high temperatures, high pressures or, in hostile environments, in combustion chambers, gas turbines, cooling steel processes, gas-quenching in furnaces,... in which either the actual method of heat and mass transfer is not known, or it cannot be assumed that the governing boundary law has a simple power law, linear Newton's law of cooling or, fourth-order Stefan-Boltzmann's radiation law. In such situations, we model this as an inverse problem of identifying a nonlinear heat transfer law at the boundary, or of the temperature-dependent heat transfer coefficient. In other fields of application, this formulation many also be considered as the concentration of gaseous diffusion with an unknown chemical reaction at surface or, as the population density with an unspecified migration law at the boundary.
We use the variational method for solving the inverse problem and we formulate the method and prove the existence result for it, as well as deliver the formula the gradient of a functional to be minimized. The numerical results provided by our method prove its efficiency.

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Generalized Quasivariational Inequality Problems and Related Problems

Bui The Hung[1]

In this paper, we apply a version of Fan-Browder's fixed point theorem to study generalized quasivariational inequality problems. Some sufficient conditions on the existence of solutions of Pareto quasi-equilibrium problems, Pareto quasivariational inclusion problems and Pareto quasi-optimization problem with multivalued mappings are shown.

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Minimax Variational Inequality and Its Application

Nguyen Quang Huy[1]

We introduce a new notion called minimax variational inequality (MVI). The solution existence of nonmonotone MVIs in Euclidean spaces, pseudomonotone MVIs in reexive Banach spaces, and strongly monotone MVIs in Hilbert spaces is studied in detail. We show that MVIs can serve as a good tool for studying minimax problems given by convex sets and differentiable functions.

This talk is based on the joint paper: N. Q. Huy and N. D. Yen, *Minimax Variational Inequalities*, Acta Math. Vietnam., Vol. 36, 265-28, 2011.

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Topological Characterizations of the Existence of Solutions to Optimization-Related Problems

Phan Quoc Khanh[1], **Nguyen Hong Quan**[2]

Necessary and sufficient conditions for the solution existence of optimization-related problems are established in pure topological settings, i.e., the problems are defined on topological spaces, without linear or other structures. First, we prove full (two-way) characterizations of the existence of solutions to variational relation problems in terms of KKM-structures and connectedness structures. Then, applying these results, we obtain such characterizations for the existence of invariant points, solutions of quasiequilibrium problems of the Stampacchia and Minty types, saddle points, and Nash equilibria for noncooperative games.

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Probable Convexity and Its Application to Data Analytics

Khoat Than[1]

Non-convex optimization problems often arise from probabilistic modeling, such as estimation of posterior distributions. Non-convexity often makes the problems intractable, and poses various obstacles for us to design efficient algorithms. In this talk, I will discuss nonconvexity from a practical viewpoint which is suitable for various problems in data science. I will discuss the concept of probable convexity which reveals the convexity of real functions in practice. This new concept is general enough that could be used in many contexts which is beyond probabilistic modeling. I then discuss an application of the concept to some problems in data analytics.

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Second-order Necessary Optimality Conditions for a Class of Semilinear Elliptic Optimal Control Problems with Mixed Pointwise Constraints

Bui Trong Kien[1], Vu Huu Nhu[2]

In this report we consider the problem of finding a control function $u \in L^p(\Omega)$ with $1 < p < +1$ and a corresponding state $y \in W_0^{1,r}(\Omega)$ which

$$\text{minimize} \quad J(y;u) = \int_\Omega L\big(x, y(x),\, u(x)\big)dx, \tag{1}$$

subject to

$$-\sum_{i,j=1}^{N} D_j\big(a_{ij}(x) D_i y\big) + h(x,y) = u \quad \text{in} \ \ \Omega \tag{2}$$

$$y = 0 \ \ \text{on } \Gamma \tag{3}$$

$$g_i\big(x, y(x), u(x)\big) \in Q_i(x) \ \text{a.e.}\ x \in \Omega,\ i = 1,2,\ldots,m. \tag{4}$$

where $L : \Omega \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and $h : \Omega \times \mathbb{R} \to \mathbb{R}$ are Carathéodory functions, $Q_i : \Omega \rightrightarrows \overline{\mathbb{R}}$ are measurable multifunctions such that for each $x \in \Omega$, $Q_i(x)$ are nonempty closed convex sets, and $g_i : \Omega \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ are given functions.

Under the generalized regularity condition which is based on adjacent tangent cones and generalizes the Robinson constraint qualification condition, we derive the first- and second-order optimality conditions for problem (1)-(4).

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An Application of Yamada-Watanabe Approximation to Study the Strong Rates of Convergence of the Approximation for Stochastic Differential Equations

Ngo Hoang Long[1]

In a seminal paper, Yamada and Watanabe [4] developed an approximation method to study the existence and uniqueness of stochastic differential equation with non-Lipschitz coefficients. In this talk, we apply their method to study the strong rates of convergence of some types of Euler-Maruyama approximation for stochastic differential equations with irregular coefficients. Our findings extend recent results in [1, 2, 3].

This talk is based on several joint works with Luong Duc Trong (HNUE) and Dai Taguchi (Ritsumeikan University).

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Generic Sigularities of the Attainable Set on Surfaces with Boundary

Hy Duc Manh[1]

A control system is given by a smooth family of vector fields parametrized by a control parameter whose range is a closed smooth compact manifold of finite dimension or a disjoint union ofsuch manifolds (possibly, with different dimensions). A *generic* system or a system in *generalposition* is a system in an open everywhere dense set in the space of systems equipped with the (sufficiently) smooth Whitney topology. Generic singularities of the field of limiting directions and the attainable set on a smooth compact surface without boundary were studied in [1]–[3]. Singularities of fields of limiting directions of generic smooth dynamical inequalities on surfaces were studied in [4]. For surfaces with generic boundary singularities of the field of limiting directions were studied in [5], and generic singularities of the positive orbit of the closed starting set which lie in the interior of this orbit were investigated in [6].

In this report, we classify singularities of attainable sets of generic systems on a smoothcompact orientable surface with boundary in the case where the starting set is a smoothlyembedded curve.

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So sánh giữa hai phương pháp rút gọn: Balanced truncation và Modal truncation

Chu Bình Minh[1], Hà Bình Minh[2]

Trong báo cáo này, chúng tôi so sánh sai số của hai phương pháp rút gọn cho hệ tuyến tính liên tục: Phương pháp balanced truncation [1] và phương pháp modal truncation [2,3]. Kỹ thuật được chúng tôi sử dụng để so sánh là các bất đẳng thức trong lý thuyết ma trận. Các kết quả so sánh sẽ được minh họa bằng các ví dụ số.

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Robust Finite-Time Stability of Linear Differential-Algebraic Equations with Time-Varing Delays

<u>Nguyen Huyen Muoi</u>[1], M. V. Bulatov[2], Vu Ngoc Phat[1]

This paper deals with the problem of robust finite-time stability for linear differential-algebraic equations with time-varying delays. Based on Lyapunov-Krasovskii function method and linear matrix inequality (LMI) technique , new sufficient conditions formulated in terms of LMI which ensure the robust finite-time stability of the system are given. An numerical example is given to illustrate the efficiency of the proposed methods.

Key words: Differential-algebraic equation, finite-time stability, time-varying delay, Lyapunov function, linear matrix inequality.

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A Splitting Method for a Backward Parabolic Equation with Time-Dependent Coefficients

Nguyen Thi Ngoc Oanh[1]

In this report, we propose a stable numerical method for an ill-posed backward parabolic equation with time-dependent coefficients in a parallelepiped. The problem is reformulated as an ill-posed least squares problem which is solved by the conjugate gradient method. To enhance the stability and the accuracy of the numerical solution to the problem we apply this approach to the discretized optimization problem rather than to the continuous one. The difficulties with large dimensions of discretized problems are overcome by using a splitting method which only requires the solution of easy-to-solve one-dimensional problems. The numerical results provided by our method prove its efficiency.

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Solving Parameter-Dependent Lyapunov Equations Using Reduced Basis Method

Nguyen Thanh Son[1], **Tatjana Stykel**

Our aim is to numerically solve parameter-dependent Lyapunov equations, especially those affinely depend on the parameter. Such equations arise in parametric model order reduction.

We use the reduced basis method for the problem. For Lyapunov equations with symmetric positive definite matrix coefficients, we derive an a posteriori error estimate using min-θ approach. With this error estimate, a Greedy algorithm for (offline) constructing the reduced basis is formulated. Numerical examples are presented.

Key words: Parameter-dependent Lyapunov equations, affine dependence, reduced basis method, θ-min approach, a posteriori error estimate, Greedy algorithm.

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Solving Orienteering Problem via DCA Branch and Cut

Ta Anh Son[2], Le Thi Hoai An[1], Ha Trong Sy[2]

Orienteering problem is well-known as a NP-hard problem with many applications in the field of transportation. This problem aims to find a path between a given set of control points (where the points of source and destination are specified) while maximizing the total score of points collected and satisfying the distance constraint. In this paper, we first analyze the structure of a generalized orienting problem which is in fact a Mixed Integer Linear Programming problem (MILP) and then propose two algorithms, based on DC (Difference of Convex functions) programming and DCA (DC Algorithms), for its solution. The first is a DCA CUT method recently introduced for MILP problems. The second is a combined DCA CUT - Branch and Bound algorithm. Preliminary numerical experiments are reported which show the efficiency of the proposed algorithm.

Keywords: Orienteering problem, DC programming, DC Algorithm, DCA Cut algorithm, Mixed 0-1 program.

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Metric Regularity in Variational Analysis Characterizations and Stability Theory

Nguyễn Hữu Trọn[1]

In this report, we present some problems on Metric regularity of mappings/set-valued mappings, simultaneously give some characterizations and study perturbation stability of the metric regularity of multifunctions on complete metric spaces. These results are based the recent paper by Ngai, Tron, Théra, Metric regularity of the sum of multifunctions and Applications, published online, august 2013, JOTA.

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Convergence of DCA Iterative Sequences in Quadratic Programming

Hoang Ngoc Tuan[1]

By using error bounds for affine variational inequalities we prove that any iterative sequence generated by *the Projection DC* (Difference-of-Convex functions) *decomposition algorithm in quadratic programming is R-linearly convergent*, provided that the original problem has solutions. Our result solves in the affirmative the first part of the Conjecture stated by Le Thi, Pham Dinh and Yen in their recent paper: Properties of two DC algorithms in quadratic programming, *J. Global Optim.* 49, 481-495 (2011).

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Một phương pháp xấp xỉ trong giải bài toán quy hoạch phân tuyến tính

Nguyễn Anh Tuấn[1], **Nguyễn Văn Quý**[2], **Nguyễn Vũ Nhân**[3], **Bùi Quốc Độ**[4]

Báo cáo trình bày một thuật toán hữu hạn bước giải bài toán quy hoạch phân tuyến tính với miền ràng buộc là hệ bất phương trình tuyến tính.

Chúng ta đã biết bài toán quy hoạch phân tuyến tính có hàm mục tiêu là hàm đơn điệu trên các đoạn thẳng. nửa đường thẳng, đường thẳng bất kỳ thuộc miền xác định của nó trong $\mathbb{R}^n$. Vì vậy, có thể xây dựng được các thuật toán kiểu đơn hình để giải bài toán. Hiện nay, hầu hết các thuật toán kiểu đơn hình sử dụng để giải các bài toán quy hoạch tuyến tính hay phân tuyến tính đều phải đưa bài toán về dạng chính tắc, tức là bài toán với miền ràng buộc là hệ phương trình tuyến tính và các biến không âm. Như vậy đã làm cho bài toán tăng số biến hay nói đúng hơn là bài toán gốc đã được giải qua một bài toán trung gian có số chiều lớn hơn.

Thuật toán đề nghị trong báo cáo này thuộc lược đồ xấp xỉ trong kiểu đơn hình, tuy nhiên vì sử dụng khái niệm nón xoay nên nó đã giải trực tiếp bài toán quy hoạch phân tuyến tính với miền ràng buộc là hệ bất phương trình tuyến tính. Trong trường hợp đặc biệt khi mẫu số của hàm mục tiêu bài toán đồng nhất bằng 1, thì thuật toán trở thành một thuật toán giải bài toán quy hoạch tuyến tính dạng chuẩn khá đơn giản và thuận tiện.

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Monotonic Optimization: Recent Developments and Applications in Wireless Communication

Hoang Tuy[1]

In recent years there has been a growing interest to Monotonic Optimization due to its many engineering applications. In this talk we will review some recent theoretical and algorithmic advances in Monotonic Optimization and also some important applications of Monotonic Optimization in wireless communication.

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Sự hội tụ của tập nghiệm Pareto tương đối

Nguyễn Văn Tuyên[1]

Khái niệm nghiệm cực tiểu Pareto tương đối được đưa ra bởi Trương Quang Bảo và B. S. Mordukhovich [1]. Trong đó, phần trong của nón thứ tự được thay thế bằng một số dạng phần trong tương đối.

Giả sử $A_n, n = 1,2,...$ là một dãy các tập con khác trống trong không gian Banach Z và $C_n, n = 1,2,...$ là các nón lồi của Z. Tập các nghiệm cực tiểu Pareto tương đối của A_n tương ứng với nón C_n được kí hiệu là $\operatorname{Re}\operatorname{Min}\left(A_n \middle| C_n\right)$. Trong báo cáo này, chúng tôi sẽ đưa ra các điều kiện thích hợp để từ sự hội tụ của A_n và C_n tương ứng tới A và C suy ra dãy tập $\operatorname{Re}\operatorname{Min}\left(A_n \middle| C_n\right)$ hội tụ tới $\operatorname{Re}\operatorname{Min}\left(A \middle| C\right)$.

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