

# Adaptive One-Step Ahead Control of Systems with Unstable Discrete-Time Inverse

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**Abstract:** Indirect adaptive control of discrete time systems was developed around 1980. It uses real time data to update the coefficients of a difference equation model after each new feedback measurement is received, using the Projection Algorithm or Recursive Least Squares. Then based on the current model, the control action needed in the current time step in order that the output in the next sample time is on the desired trajectory is computed, and applied to the system. This is termed, one-step ahead control. The theory developed could only be applied to systems with discrete-time transfer functions that have a stable inverse, i.e. have all zeros inside the unit circle.

This condition very severely limits the applicability of the method. When a continuous time system fed by a zero-order hold, is converted to a difference equation whose solution is identical to that of the original differential equation, zeros are introduced. For any system with at least 3 more poles than zeros, there are so-called sampling zeros introduced that are outside the unit circle, and the adaptive control method cannot be applied. It is the purpose of this paper to develop a new indirect adaptive discrete time control theory that applies in general.

The one-step ahead control computation is equivalent to finding the system inverse, which is unstable when there are zeros outside the unit circle. New results from our research group give a method to obtain a stable inverse of a system whose normal inverse is unstable. There are several approaches to doing this, but what is used here employs an increased sample rate. Given the desired trajectory at original sample times, one additional time step is introduced between each sample time for 3rd order poles excess. For 5th order pole excess we introduce two extra time steps. The control action producing zero error at the original time steps can be computed using a procedure related to pseudo inverse. And it is proved that the control actions requested are stable.

This addresses the unstable inverse issue. Then to implement adaptation, we make use of the Model Predictive Control (MPC) model formulation, and update the matrices with a projection algorithm recursively in real time, and update a pseudo inverse. The original indirect adaptive control algorithm for discrete time systems, only works for a very restricted set of systems in the world, mostly limited to first and second order systems. This new algorithm allows indirect adaptive control to apply to general time invariant linear discrete time systems.

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