Uncertainty Quantification using Parallel Multi-level Monte Carlo: Applications to Shallow Water, Euler, Magnetohydrodynamics, and Multi-phase Cavitation Flows

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Abstract: Complex liquid, gas, and plasma flow problems can be modeled in terms of nonlinear conservation laws. Examples include shallow water equations for lakes, rivers, earthquake or landslide generated tsunamis, multi-phase Euler equations for cavitating vapor cloud collapses, and Darcys law for porous subsurface flows. Many of the above non-linear dynamical systems exhibit strong dependence on uncertain input data, such as initial data, sources and model coefficients. In this talk I will present a mathematical setting as well as two numerical methods for non-intrusive uncertainty quantification and propagation in such flows, namely finite volume method for the spatio-temporal discretization and the multi-level Monte Carlo statistical sampling technique, which accelerates standard Monte Carlo method by several orders of magnitude using clever variance reduction obtained from simulations with coarser spatio-temporal resolutions as control variates. Efficient implementation of such hierarchical discretization schemes relies on several more advanced mathematical concepts such as multi-level alias-free representation for random bathymetry and unbiased spectral parallel FFT generation of random porosity fields. Numerical experiments using in-house developed ALSVID-UQ, Cubism-MPCF, and PyMLMC software up to one trillion mesh elements and 500'000 cores will be presented, illustrating the efficiency of the methods and pushing be boundaries of current scientific knowledge in this field.

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