Asymptotic Controllability and Infinite Horizon Optimal Control – A Weighted Sobolev Space Approach

S. Pickenhain[1] **and V. Lykina**[2]

Abstract: We address an infinite horizon optimal control problem in Hilbert spaces to design a control which is asymptotically stabilizing a nonlinear control system.

The concept of designing a feedback controller such that an integral of the square of the tracking error over an infinite horizon is minimized was first proposed by Wiener 1943 and Hall 1949. The conventional theory of the regulator problem in the papers of Wiener and Hall is based on Fourier and Laplace transforms and is restricted to the case of autonomous problems. In 1960, Kalman proposed another approach to the problem: he formulated a receding horizon regulator problem to design a feedback control law.

However, the integration over an unbounded time interval indicates some difficulties which are specific to this class of problems. Such problematic features contain, for instance, the lack of standard transversality condition which results in a missing boundary condition in the system of necessary optimality conditions. This, in turn, leads to extreme sensitivity of numerical procedures making it even impossible to find a solution. Another substantial problem is the non-compactness of the time domain, which makes serious problems in the proofs of existence results due to non-extendible embedding properties of the underlying spaces onto the unbounded domain.

For this reason, one needs other techniques to cope with typical difficulties arising in infinite horizon control problems. The suggestion of introducing Weighted Sobolev spaces as state spaces (and Hilbert spaces) was made by the authors. It holds many interesting effects and advantages both for the modeling itself and the theoretical and numerical treatment of the control problem. The systematic analysis and discussion of these effects is demonstrated by means of examples and builds the main issue of this paper.

[1,2] Mathematical Institute, Chair of Optimization
Brandenburg University of Technology at Cottbus
P.O.B. 10 13 44, D-03013 Cottbus, Germany
sabine.pickenhain@tu-cottbus.de, Valeriya.Lykina@b-tu.de