Theory and Application of Time Delayed Optimal Control Problems with State Constraints

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Abstract: Time delays often appear in real-world engineering, chemical and biological systems that are modelled by differential equations. Time delays also play an important role in communication and information technologies. In this talk, we consider optimal control problems with time delays in state and control variables. The control system is subject to control and state constraints. Using the methods of steps, which was applied in [1] for control problems with mixed control-state constraint, we can derive the necessary optimality conditions (Maximum Principle) for control problem with pure state constraints. Under certain regularity conditions (see the report [3] in the non-delayed case), the measure is shown to have a density on boundary arcs. We note that a Maximum Principle for non-smooth optimal control problems with time delays and state constraints has recently been presented by Vinter [4]. We extend the numerical methods in [1] to solve time-delayed optimal control with state constraints by discretization and nonlinear programming methods. The discretization approach allows us to compute the multiplier associated with the state constraints and to check the necessary optimality conditions. As a motivational example we discuss cancer therapies combining anti-angiogenic treatment with chemotherapy. The numerical approach is further illustrated by two examples: (1) the optimal control of a two-tank continuous stirred tank reactor (CSTR) with a temperature constraint and (2) a delay differential model of immuno-chemotherapy of cancer with time delays in control and state variables [2], where a state constraint is imposed on the healthy cells. Parts of the talk are joint work with L. Göllmann and A. Swierniak.

References

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