The Compressible Euler Equations with Gravity: Well-balanced Schemes and all Mach Number Solvers

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Abstract: We consider astrophysical systems that are modeled by the multidimensional Euler equations with gravity.

First for the homogeneous Euler equations we look at flow in the low Mach number regime. Here for conventional finite volume discretizations one has excessive dissipation in this regime. We identify inconsistent scaling for low Mach numbers of the numerical fux function as the origin of this problem. Based on the Roe solver a technique that allows to correctly represent low Mach number flows with a discretization of the compressible Euler equations is proposed. We analyze properties of this scheme and demonstrate that its limit yields a discretization of the incompressible limit system.

Next for the Euler equations with gravity we seek well-balanced methods. We describe a numerical discretization of the compressible Euler equations with a gravitational potential. A pertinent feature of the solutions to these inhomogeneous equations is the special case of stationary solutions with zero velocity, described by a nonlinear PDE, whose solutions are called hydrostatic equilibria. We present well-balanced methods, for which we can ensure robustness, accuracy and stability, since it satisfies discrete entropy inequalities.

We will then present work in progress where we combine the two methods above.

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