Domain Decomposition Method for Two-Dimensional Parabolic Problems

Y. Jun[1]

Abstract: Domain decomposition (DD) methods are often used for solving partial differential equations (PDE). They are very efficient, especially when a parallel computer is used. The basic idea of the DD method to solve the PDE is that the original spatial domain is decomposed into subdomains and the PDE on each subdomain is solved in parallel manner. In the spatial domain, the problem is decomposed in stripwise or rectangular manner with overlapping subdomains or non-overlapping subdomains.

In this paper, we consider the two-dimensional parabolic partial differential equation of the form

$$u_t = u_{xx} + u_{yy} + \alpha(x,y)u_x + \beta(x,y)u_y + \gamma(x,y)u + f(x,y,t) \tag{8}$$

defined in the unit square $\Omega = [0,1] \times [0,1]$ and $0 \leq t \leq T$. We focus our attention on the use of non-overlapping stripwise decomposition to solve Equation (8). The non-overlapping decomposition is defined so that the original spatial domain is decomposed into independent subdomains in stripwise manner in which two adjacent subdomains share one interface line without overlapping any region. In order to solve the PDE on each subdomains concurrently in a parallel computing environment, the values on the interface line must be estimated in advance.

In this paper, an efficient and unconditionally stable non-overlapping stripwise domain decomposition method is presented. The order of accuracy of the interface lines and the interior regions of the method is second. Numerical experiments are presented to support efficiency, accuracy, and unconditional stability of the method.

[1] Department of Applied Mathematics
Kumoh National Institute of Technology
Gumi, Gyeongbuk 39177, South Korea
yjun@kumoh.ac.kr