# Determination of the Initial Condition
# in Parabolic Equations

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**Abstract:** The problems of reconstructing the initial condition in parabolic equations from the observation at the final time, from integral observations regarded as generalization of interior observations, and from the boundary observation are presented and analyzed. These inverse problems are reformulated as variational problems which minimize appropriate misfit functionals. It is proved that these functionals are Fréchet differentiable and the formula for their gradient via adjoint problems is derived. The direct problems have been first discretized in space variables by the finite difference method or the finite element method and the variational problems have been correspondingly discretized. The convergence of the solution of the discretized variational problems to the solution of the continuous ones is proved. To solve the problems numerically, we further discretize them in time. It is proved that the completely discretized functionals are Fréchet differentiable and the formulas for their gradient have been derived via discrete adjoint problems. The problems are then solved by the conjugate gradient method and tested on computer. Also, as a by-product of the variational method, based on Lanczos' algorithm, we suggest a simple method to approximate the degree of ill-posedness.

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