

# Model Predictive Control of Stochastic Processes via Probability Density Functions

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**Abstract:** When viewing stochastic processes from a statistical perspective, the problem of controlling the process in an optimal way can often be rendered deterministic by characterizing the state by the underlying probability density function (PDF). For stochastic processes that are modeled by Itô stochastic differential equations, under mild assumptions, the evolution of the associated PDF is described by the Fokker–Planck (FP) equation, a second order parabolic PDE. In this manner, optimal control of the stochastic process can be achieved by solving an optimal control problem subject to the FP equation. A Model Predictive Control (MPC) scheme is then applied to track the solution of this equation over a fixed time horizon.

In this talk, we take a closer look at the stability of the MPC closed loop feedback system. Avoiding the use of stabilizing terminal costs or constraints, we study the prediction horizon length that is needed to guarantee stability. A useful property to this end is the exponential controllability assumption. Furthermore, we modify the MPC stage cost to analyze qualitative changes in the horizon length for various parameters in the problem setting, putting emphasis on the case of instantaneous control.

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