Binary Programming with Semilinear Elliptic PDE-constraints

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Abstract: We present an outer approximation algorithm for binary programming problems where the constraints arise implicitely from semilinear elliptic PDEs and bounds on the states. A typical example is the heating of a metallic workpiece by a given finite set of heat sources which may be switched on or off, with the objective of using as few sources as possible in order to achieve a given minimum temperature everywhere in the workpiece.

In the case of linear PDEs, many such problems can be rewritten as (finite-dimensional) linear or convex quadratic integer programs over the controls, and hence solved by state-of-the-art integer programming software. For the non-linear case, the standard solution approach is to directly discretize the entire problem, resulting however in huge non-convex mixed-integer optimization problems that can be solved to proven optimality only in very small dimensions. For PDEs with a convex nonlinear part, which appear in the application mentioned above, we show that the solution operator is pointwise concave and submodular. This allows to over- and underestimate it by linear expressions, leading to linear cutting planes that can be used to cut off any infeasbile solution. By a suitable combination of such over- and underestimators, our approach can also handle linear constraints on both control and state variables as well as L^p-tracking-type objective functions for all $p \in [1, \infty]$.

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