Solving Standard Quadratic Programming by Cutting Planes

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Abstract: We focus on the generation of new cutting planes (cuts for short) for the general set

$$\Gamma_{a,b} = \{(x, Y) \in \mathbb{R}^d \times \mathbb{R}^{d \times d} \mid Y = xx^T, x \in \Delta_{a,b}\},$$

where $\Delta_{a,b} = \{x \in \mathbb{R}^d \mid \sum_{i=1}^{d} a_i x_i = b, x \geq 0\}$, $a \in \mathbb{R}^d_+$ and $b \in \mathbb{R}_+$. This structure appears in the solution through Spatial Branch&Bound of quadratic programs with linear constraints and in the solution of $Standard\ Quadratic\ Programs$

$$\min\{x^T Q x \mid x \in \Delta_{e,1}\},$$

where $Q \in \mathbb{R}^d \times \mathbb{R}^d$ and $\Delta_{e,1}$ is the $standard\ simplex$ as an important special case.

We concentrate on cutting planes for $\Gamma_{e,1}$ which by simple scaling considerations can be transformed into cutting planes for $\Gamma_{a,b}$. Because this set is non-convex, we are practically interested in its classical convex relaxation obtained by replacing constraints $Y = xx^T$ with the linear McCormick estimators. In addition, it is well known that to obtain even stronger convex relaxations of $\Gamma_{e,1}$ one can use the so-called $Reformulation\text{-}Linearization\ Technique$ (RLT).

Although $\Gamma_{e,1}$ is a purely continuous set it has strong connections with the maximum clique problem by a remarkable result of Motzkin and Straus. This allows us to formulate valid cutting planes and in particular cuts that correspond to an underlying complete bipartite graph structure. We study the relation between these cuts and the classical ones obtained by the first level of the reformulation-linearization technique. By studying this relation, we derive a new type of valid inequalities that generalize both types of cuts and are stronger.

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