Collocation – a Powerful Tool for Solving Singular ODEs and DAEs

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Abstract: We deal with boundary value problems for systems of ordinary differential equations with singularities. Typically, such problems have the form

$$z'(t) = F(t, z(t)), \quad t \in (0, 1], \quad B_0 z(0) + B_1 z(1) = \beta,$$

where $\lim_{t\to 0} F(t, z(t)) = \infty$ and $\lim_{t\to 0} \partial F(t, z)/\partial z = \infty$. The analysis is usually done for the model equation

$$z'(t) = \frac{1}{t^\alpha} M z(t) + f(t, z(t)), \quad t \in (0, 1], \quad B_0 z(0) + B_1 z(1) = \beta,$$

where $f(t, z)$ may also be in the form of $g(t, z)/t$ with a smooth function $g(t, z)$. For $\alpha = 1$ the problem has a *singularity of the first kind*, while for $\alpha > 1$ the singularity is commonly referred to as *essential singularity*. We briefly recapitulate the analytical properties of the above problems with a special focus on the most general boundary conditions which guarantee their well-posedness.

To compute the numerical approximation for z we use polynomial collocation, because the method retains its hight order even in case of singularities. The usual high-order superconvergence at the mesh points does not hold in general, however, the uniform superconvergence is preserved (up to logarithmic factors). We will discuss how the collocation performs for problems with unsmooth inhomogeneity $g(t, z)/t$ [1].

Recently, we have implemented the MATLAB code `bvpsuite` [2] with the special focus on the above problem class. For higher efficiency, we provided an estimate of the global error and adaptive mesh selection. The code can be applied to arbitrary order problems also in implicit formulation. Also, parameter-dependent problems and systems of index 1 differential-algebraic equations (DAEs) are in the scope of the code. We present current work on the mesh selection strategy implemented to enhance the efficiency and illustrate the performance of the software by means of numerical simulation of models in applications and DAEs of higher index.

[1] Rachůnková I., Staněk, S., Vampolová, J., Weinmüller, E.B., On linear ODEs with a time singularity of the first kind and unsmooth inhomogeneity. Boundary Value Problems, to appear.

[2] Kitzhofer, G., Koch O., Pulverer, G., Simon, Ch., Weinmüller, E.B., The New MATLAB Code for the Solution of Singular Implicit BVPs. *JNAIAM J. Numer. Anal. Indust. Appl. Math.*, 5 (2010), 113–134. Available from `http://www.math.tuwien.ac.at/~ewa`.

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