Numerical and Parallel Efficiency of Sparse-Aitken Schwarz Solver for Large-scale Groundwater Flow Problems

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Abstract: Domain decomposition methods are nowadays commonly used to solve large-scale problems in parallel. For instance, it is relevant to use the Schwarz alternating method when classical linear parallel solvers, such as Krylov or multigrid methods, do not provide a satisfactory weak scaling anymore. When a linear solver reaches its limit in terms of size, the size of the problem can still be increased by an order of magnitude using the Schwarz method that combines a few instances of the solver. In this paper, we consider the linear Darcy equation $\nabla.K(x,y,z)\nabla u(x,y,z)=0$ with suitable boundary conditions, where the permeability field K is generated randomly according to a log-normal distribution. Unfortunately, the convergence of domain decomposition methods such as the Schwarz alternating method can be very slow for this problem. In order to overcome this problem, it has been proposed in [1] to use the Aitken's acceleration formula in a low-rank space. The low-rank space is computed from the singular value decomposition of the successive boundary conditions that are exchanged during Schwarz iterations. This Aitken-Schwarz solver is purely algebraic because it does not require any informations about the underlaying equations or their discretization. Preliminary results were also given in [2] for a Fortran code with a multigrid method as local solver. We now propose an MPI implementation that uses the PETSc library to solve local problems. Therefore, there are two levels of parallelism. This implementation allows us to study and improve the numerical and the parallel efficiency of the method for different local solvers provided by the PETSc library. We also enhance the computing of the acceleration with preserving the sparsity of the error operator involved in the Aitken acceleration. We present numerical results up to one billion unknowns in 3D, discussing the efficiency and the robustness of the method when the variance and the correlation length of the distribution vary.

1 D. Tromeur-Dervout, *Meshfree Adaptative Aitken-Schwarz Domain Decomposition with application to Darcy Flow*, Parallel Distributed and Grid Computing for Engineering, Computational Science Engineering & Technology,volume 21,pages 217–250, 2009

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