Enlargements and Autoconjugate Representations of Maximally Monotone Operators

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Abstract: The aim of this talk consists in presenting new results on enlargements as well as on autoconjugate representations of maximally monotone operators. More precisely, let X be a real Banach space with continuous dual $X^\star$. We consider a generalized equation governed by a maximally monotone operator T from X into the subsets of $X^\star$, that is the problem of finding $x \in X$ such that $0 \in T(x)$. This model has been extensively used as a mathematical formulation of fundamental problems in optimization and fixed point theory.

Solving the previous inclusion is tantamount to finding a point of the form $(x, 0)$ in the graph of T. When T is set-valued, the problem could be ill-behaved, making the required computations hard. Enlargements of T are point-to-set mappings (the terms set-valued mapping and multifunction are also used) which have a graph larger than the graph of T. These mappings, however, have better continuity properties than T itself. Moreover, they stay "close" to T, so they allow to define perturbations of the initial problem, without losing information on T. In this way, we can define well-behaved approximations of the initial problem, which (i) are numerically more robust, and (ii) whose solutions approximate accurately the solutions of $0 \in T(x)$. The use of enlargements in the study of such a problem has been a fruitful approach, from both practical and theoretical reasons.

One key ingredient for this study is the (variational) representation of operators as introduced independently by Fitzpatrick et Martinez-Legaz & Théra.

It is nowadays well established that:

- each representable operator is monotone;
- each monotone operator is not necessarily representable;
- each maximally monotone operator is representable;
- a representable operator is not necessarily maximally monotone.

In other words, the family of representable operators forms a class strictly included in between monotone operators and maximally monotone operators.

A typical example of representable operator is given by the subdifferential of a lower semicontinuous proper function f. It can be observed that it is represented by the Fenchel-Young mapping: $g(x, x^\star) = f(x) + f^\star(x^\star)$ associated to f, where $f^\star$ stands for the Fenchel conjugate of f. It should be noticed that this representation is auto-conjugate in a sense which will be defined during the lecture. We will also give some recent developments on auto-conjugate representations of maximally monotone operators.

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