

# Building and Testing Well-balanced Numerical Schemes for a Model of Two-phase Flows

M. D. Thanh<sup>1</sup>

**Abstract:** In this talk we will present a family of numerical schemes for approximating solutions of a model of two-phase flows. The governing system of equations is arisen from the modeling of deflagration-to-detonation transition in granular materials, and has the form of a hyperbolic system of balance laws in nonconservative form. The construction of the family of numerical schemes for this model consists of several stages. In the first stage we absorb the source term in nonconservative form into equilibria. The second stage is designed to incorporate the equilibria obtained from the first stage into a general numerical flux of conservation laws. Moreover, we also define a family of such numerical fluxes by taking convex combinations of the numerical flux of a stable such as the Lax-Friedrichs scheme and the one of a higher-order scheme such as the Richtmyer scheme. Tests show that numerical schemes obtained in this family possess interesting property: they are fast and stable, and they are well-balanced. These schemes are shown to give a very good accuracy.

## References

- [1] D. Kröner and M.D. Thanh, Numerical solutions to compressible flows in a nozzle with variable cross-section. *SIAM J. Numer. Anal.*, 43 (2005), 796–824.
- [2] P.G. LeFloch and M.D. Thanh. A Godunov-type method for the shallow water equations with variable topography in the resonant regime. *J. Comput. Phys.*, 230 (2011), 7631–7660.
- [3] M.D. Thanh, D. Kröner, and N.T. Nam, Numerical approximation for a Baer-Nunziato model of two-phase flows, *Appl. Numer. Math.*, 61 (2011), 702–721.
- [4] M.D. Thanh, D. Kröner, C. Chalons, A robust numerical method for approximating solutions of a model of two-phase flows and its properties, *Appl. Math. Comput.*, 61 (2011), 702-721.
- [5] M.D. Thanh, A well-balanced Roe-type numerical scheme for a model of two-phase flows, *J. Korean Math. Soc.*, 51 (2014), 163-187.
- [6] M.D. Thanh, Building fast well-balanced two-stage numerical schemes for a model of two-phase flows, *Commun. Nonl. Sci. Num. Simulat.*, 19 (2014), 1836-1858.
- [7] M.D. Thanh, A phase decomposition approach and the Riemann problem for a model of two-phase flows, *J. Math. Anal. Appl.*, 418 (2014), 569-594.

---

<sup>1</sup> Department of Mathematics, International University, Quarter 6, Linh Trung Ward, Thu Duc District, Ho Chi Minh City, Vietnam [mdthanh@hcmiu.edu.vn](mailto:mdthanh@hcmiu.edu.vn)