Asymptotics of Randomly Weighted Sums of Dependent Heavy Tailed Random Variables with Regular Variation

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Abstract: Let$\{X_i\}$ be a sequence of Weakly Negatively dependent(WND) random variables with regularly varying tails. Let $\{W_i\}$ be a sequence of non-negative random variables, independent of $\{X_i\}$. We denote the weighted random sums $S_N = \sum_{i=1}^{N} W_i X_i$, and the tail probability of maximum of sums $\overline{S}_N = \max_{k \leq N} \sum_{i=1}^{k} W_i X_i$ where N is a non-negative integer valued random variable. Under the assumption that $\{X_i\}$, $\{W_i\}$ and N are mutually independent with some mild conditions, this paper establishes an asymptotic relationship for the tail probability $P(\overline{S}_N > x)$.

Applications to Insurance Risk

Randomly weighted sums and their maxima are often dealt in ruin probabilities of discrete time risk models. Let the net losses during reference period $\{X_i\}_{i\geq 1}$ constitute a sequence of identically distributed but WND random variables. The surplus is invested into risk assets by the insurance company. Let $\{Y_i\}_{i\geq 1}$ be a sequence of non-negative random variables representing discount factors from time i to $i-1$, which are independent of $\{X_i\}$. Let $W_n = \prod_{j=1}^{n} Y_j$ be the discount factor from the year n to time 0. If the insurance company starts with an initial capital x, then the discounted surplus after n years is given by

$$U_n = x - \sum_{i=1}^{n} W_i X_i, \quad n \geq 1$$

Then the ruin probability at the end of the year n is defined as

$$\psi(x, n) = P(\overline{S}_n > x), \quad n \geq 1.$$

It is reasonable to assume that the net losses follow some dependent structure and the return rates may also be dependent. Our results can be applied to the finite time ruin probabilities of discrete time risk models with dependent structures in net losses and returns.

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