Pontryagins Maximum Principle for Infinite Horizon Optimal Control Problems with State Constraints

N. Tauchnitz[1]

Abstract: Optimal control problems with infinite horizon arise in many fields, in particular in the economic growth theory. Typically, the performance index is described by an integral on an unbounded interval. In this talk we present the Pontryagins maximum principle for the infinite horizon optimal control problem with state constraints:
Minimize the functional

$$J\big(x(\cdot), u(\cdot)\big) = \text{(L-)}\int_0^\infty f\big(t, x(t), u(t)\big)e^{-\varrho t}\, dt \to \inf$$

(herein (L-) $\int$ denotes the Lebesgue-integral) subject to the state equation

$$\dot{x}(t) = \varphi\big(t, x(t), u(t)\big), \quad x(0) = x_0,$$

the state constraints

$$g_j\big(t, x(t)\big) \leq 0 \ \ \forall t \in \mathbb{R}_+, \quad j = 1, ..., l,$$

and control restrictions

$$u(t) \in U \subseteq \mathbb{R}^m, \quad U \neq \emptyset.$$

We consider this optimal control problem for measureable and bounded controls $u(\cdot)$ and for states $x(\cdot)$ which belong to the Weighted Sobolev space $W_2^1(\mathbb{R}_+, \mathbb{R}^n; \nu)$:

$$W_2^1(\mathbb{R}_+, \mathbb{R}^n; \nu) := \Big\{x(\cdot) \,\Big|\, \int_0^\infty \big(\|x(t)\|^2 + \|\dot{x}(t)\|\big)\nu(t)\, dt < \infty\Big\}, \ \ \nu(t) = e^{-at},\, a > 0.$$

For the class of problems proposed, we state the Pontryagin maximum principle and give applications. The obtained Pontryagin maximum principle includes the adjoint equation, the maximum condition and also transversality conditions.

[1] Department of Mathematics
BTU Cottbus
LS Optimierung
PF 10 13 44
03013 Cottbus, Germany
tauchnitz@math.tu-cottbus.de