Energy Conservation in Subway Operation by Optimal Scheduling

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Abstract: Energy optimal operation of cam controlled subway trains requires the numerical solution of nonlinear optimal control problems with both continuous and integer controls and with various point constraints. The paper describes rigorous mathematical solution approaches and gives numerical results for optimal operation.

The indirect approach based on Pontryagin's Maximum Principle and the Competing Hamiltonians algorithm developed by the authors leads to intricate multi-point boundary value problems in state and adjoint variables with jumps and switching conditions with difficult stability problems.

As principal alternative, a direct approach based on outer convexification, relaxation and the Krein-Milman theorem allows for offline solution of mixed integer control problems with no integer gap while avoiding the combinatorial explosion of computing time. Moreover, arbitrarily good approximations by integer solutions with finitely many switches can be constructed by adequate rounding procedures.

Advanced Multiple Shooting methods for the numerical solution of the intricate boundary value problems as well as optimization boundary value problems in both the indirect and the direct approach are described.

Applications to the energy optimal operation of subway trains in the New York subway system demonstrate the high potential for energy savings. In particular, results for a new energy optimal scheduling of station-to-station transits in a whole line are presented.

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