A Geometric Multigrid Solver for Fluid-Structure Interactions

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Abstract: We consider the coupling of the incompressible Navier-Stokes equations with an elastic solid. The fully coupled set of equations is discretized with a strictly monolithic finite element technique in the *Arbitrary Lagrangian Eulerian* framework. The resulting algebraic system of equations compirises various difficulties:

1. Nonlinearities due to complex material laws but also due to the mapping of the Navier-Stokes system to an ALE reference framework.

2. Very bad conditioning of the linearized equations due to the saddle point character and a coupling of the Navier-Stokes problem (of parabolic type) with the elastic structure system (of hyperbolic type).

3. Finally, and in particular for three dimensional problems, the sheer size of the coupled system.

The design of efficient solvers is difficult, as the structure of the coupled system does not allow for standard approaches.
In this talk, we will present a solution scheme, that is based on a fully monolithic Newton's method for linearization and a fully implicit geometric multigrid solver for the linear equations. Only in the smoother of the multigrid iteration, we apply a partitioning into three sub-problems: the solid field, the ALE mapping and the Navier-Stokes system. We will show, that by this splitting we evade the very bad conditioning of the coupled system matrix.
In a first step, we demonstrate, that a direct solution to the three sub-problems will lead to an optimal multigrid solver with convergence rates tending to zero for increasing number of mesh-levels. Second, we will show robust scaling of the fully iterative scheme, if only an approximation to three sub-problems is applied within each smoothing step.

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