Finite Element Error Estimates for Dirichlet Control Problems in Convex and Non-convex Domains

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Abstract: In this talk we study control constrained Dirichlet boundary control problems governed by an elliptic partial differential equation. We assume that the underlying domain is polygonal but not necessarily convex.

In the first part of this talk, we investigate the regularity of the solution of such problems. It is well known that in polygonal domains the solution of an elliptic partial differential equation contains singular terms in general which are due to the corners of the domain and depend on the size of the corresponding angle. By analyzing these singular parts in detail we are able to establish improved regularity results in convex domains compared to those in the literature. Moreover, we derive completely new regularity results for problems posed in non-convex domains. For example, we show that the optimal control is a continuous function although the normal derivative of the adjoint state may be unbounded.

In the second part, we discuss error estimates for the finite element approximation of the optimal control problem. We discretize both the state and the control by piecewise linear and continuous functions on quasi-uniform meshes. The error estimates which we obtain mainly depend on the size of the interior angles but also on the presence of control constraints. In case of an unconstrained control in non-convex domains, the error estimates can even be worse compared to those for control constrained problems.

Finally, different numerical examples are presented in order to illustrate the theoretical results.

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