A Priori Finite Element Discretization Error Estimates for Dirichlet Optimal Control Problems with Control and State Constraints

M. Mateos[1] and I. Neitzel[2]

Abstract: We consider an optimal control problem subject to a linear elliptic PDE-constraint and pointwise state constraints in the interior of the two-dimensional convex polygonal computational domain. The control enters the problem formulation via Dirichlet boundary conditions, and may or may not be subject to pointwise control bounds. We derive a priori error estimates for the finite element discretization of the model problem. More precisely, we obtain error estimates for the $L^2(\Gamma)$-norm of the optimal control of order $h^{3/4-2/p}$ in the control-constrained case, and $h^{1-1/p}$ in the other case. Here, p is a real number depending on the boundary. The challenge of this work is due to the fact that unlike in e.g. distributed or Neumann control problems, the state functions associated with L^2-Dirichlet controls have very low regularity, being elements of $H^{1/2}(\Omega)$. By considering the state constraints in the interior it is possible to make use of higher regularity and separate the regularity limiting influences of the boundary on the one hand, and the measure in the right-hand-side of the adjoint equation on the other hand. We note in passing that in case of control constraints, these may be interpreted as state constraints on the boundary.

[1] Departamento de Matématicas, E.P.I. Gijón, Universidad de Oviedo, Campus de Gijón, 33203 Gijón, Spain
mmateos@uniovi.es

[2] Zentrum Mathematik, Lehrstuhl M17, Technische Universität München, Boltzmannstr. 3, 85748 Garching, Germany
neitzel@ma.tum.de