The k-Edge-Connected 3-Hop-Constrained Network Design Problem Polyhedra: Integer Programming Formulations and Branch-and-Cut

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Abstract: Given an undirected edge-weighted network $G = (V, E)$, a set of pair of nodes $D \subseteq V \times V$ and two integers $k \geq 2$ and $L \geq 2$, the *k-edge-connected L-hop-constrained network design problem (kHNDP for short)* consists in finding a minimum weight subgraph of G containing at least k edge-disjoint st-paths (that is paths between s and t) for every pair of nodes $\{s, t\} \in D$. Moreover, these st-paths are required to contain no more than L edges.

The kHNDP is related to the design of telecommunication networks where operators require the network to satisfy some survivability and QoS constraints. On one hand, the survivability is ensured by the existence of k edge-disjoint paths between every pair of nodes $\{s, t\} \in D$. In case of failure of at most $k - 1$ edges of the network, there still exists a routing path between s and t. On the other hand, the QoS-constraint is ensured by the limit on the number of edges (also called *hop*) of the routing paths.

In this work, we study the problem when $L = 3$. We first present some graph transformations which allow to reduce the problem into that of finding unconstrained disjoint paths, and present integer programming formulations for the problem relying on these graph transformations. Then, we investigate the polyhedra associated with the above formulations. We introduce several classes of valid inequalities and study the conditions under which they define facets. We also discuss the separation problem associated with each of these inequalities and devise a Branch-and-Cut algorithm for solving the kHNDP. Finally, we present some computational results for the kHNDP and give some concluding remarks.

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