Numerical Solution of the Density Profile Equation Using an Integral Method

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Abstract: We discuss the numerical treatment of a nonlinear singular second order boundary value problem in ordinary differential equations, posed on an unbounded domain, which represents the density profile equation for the description of the formation of microscopic bubbles in a non-homogeneous fluid. Up to now several numerical methods have been proposed to approximate the solution of this problem, including a shooting method [1] and collocation algorithms [2]. Due to the fact that the nonlinear differential equation has a singularity at the origin and the boundary value problem is posed on an unbounded domain, the proposed approaches are complex and require a considerable computational effort.

This is the motivation for our present study, where we describe an alternative approach, based on the reduction of the original problem to an integro-differential equation. In this way, we obtain a Volterra integro-differential equation of the first kind with a singular kernel. The numerical integration of such equations is not straightforward, due to the singularity. However, in this paper we show that this equation may be accurately solved by simple product integration methods, such as the implicit Euler method and a second order method, based on the trapezoidal rule. We illustrate the proposed methods with some numerical examples.

References

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