Bilevel Programming and Price Optimization Problems

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Abstract: Consider a general pricing model involving two levels of decision-making. The upper level (leader) imposes prices on a specified set of goods or services while the lower level (follower) optimizes its own objective function, taking into account the pricing scheme of the leader. This model belongs to the class of bilevel optimization problems where both objective functions are bilinear.

In this talk, we review this class of hierarchical problems from both theoretical and algorithmic points of view and then focus on some special cases. Among others, we present complexity results, identify some polynomial cases and propose mixed integer linear formulations for those pricing problem.

In the first problem considered, tolls must be determined on a specified subset of arcs of a multicommodity transportation network. In this context the leader corresponds to the profit-maximizing owner of the network, and the follower to users travelling between nodes of the network. The users are assigned to shortest paths with respect to a generalized cost equal to the sum of the actual cost of travel plus a money equivalent of travel time.

An extension of the Network Pricing Problem is obtained by optimizing the design of the network and the set of tolls on a subset of open arcs, given that users travel on shortest paths.

The third problem is a special case of the Network Pricing Problem in which the taxable arcs are connected and form a path, as occurred in toll highways. When users travel on at most one taxable subpath, the problem can be reformulated in a Network Pricing Problem on an auxiliary clique graph. Interestingly this problem is also equivalent to that of determining optimal prices for bundles of products given that each customer will buy the bundle that maximizes her/his own utility function.

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