Optimal Control Problem for Complex Heat Transfer Model

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Abstract: Optimal control problems for models of complex heat transfer in scattering media with reflecting boundaries are of great importance in connection with engineering applications. Here, the following examples can be mentioned: modeling the heat transfer in combustion chambers and industrial furnaces, estimating the efficiency of cooling systems, predicting heat transfer in glass manufacturing, control of thermal processes in optical fiber production, etc. A considerable number of works is devoted to control problems for evolutionary models involving radiative heat transfer. There, the propagation of thermal radiation is described by a nonlocal integral-differential equation or by its local approximations. The temperature field is governed by the conventional transient heat transfer equation with additional source terms accounting for the radiative heat exchange. Nevertheless, theoretical analysis of optimal control problems for steady-state models of complex heat transfer is a poorly studied. The main difficulty here is, in addition to nonlinearities of the governing equations, the absence of appropriate energy estimates.

This talk deals with steady-state models. The problem addressed is the design of reflection properties of the boundary of a three-dimensional domain in order to optimize a cost functional, e.g. to maximize the energy outflow from the domain. The application of the P_1 approximation to the radiative heat transfer equation yields an optimal boundary multiplicative control problem for a nonlinear elliptic system. The solvability of this system is proven on the basis of new a priori estimates of solution norms. Necessary optimality conditions of first order are derived, and an analogue to the bang-bang principle of optimal control theory is obtained. Sufficient conditions ensuring the non-degeneracy of the optimality system are found in the case of heat transfer in channels.

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