Computationally Efficient Time-integration of the Navier-Stokes Equations by Automatic Stepsize and Order Selection

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Abstract: We present an hp-adaptive procedure based on the Backward Differentiation Formulas to yield efficient time integration of the incompressible Navier-Stokes equations. The stepsize selection (h-adaptivity) is based on a local error estimate and an error controller to guarantee that the numerical solution accuracy is within a prescribed user tolerance. The proposed algorithm includes a mechanism to reject and restart steps for which the preset error tolerance has not been met. The order selection (p-adaptivity) relies on the idea that low-accuracy solution can be calculated efficiently by low order time integrators while high-accuracy solution requires high order time integrators to preserve computational efficiency. Thus, the stability test, on which the order selection is based, deems a method of order p stable if there is no method of lower order that delivers the same solution accuracy for a larger stepsize. To this end, the test detects growing numerical noise in approximate solutions by assessing whether or not the successive higher order terms (with proper scaling) in their Taylor series expansions form a sequence of terms with decreasing amplitudes. This is indeed a stability test given that all time integrators produce a large amount of numerical noise in the solution error when outside their stability regions. The order selection guarantees both that (1) the used method of integration is stable and (2) the time integration procedure is computationally efficient.

The hp-algorithm is specifically tuned for incompressible flow problems for which the stability restriction of the BDF depends on the Reynolds number. The proposed method uses the variable stepsize formulation of the BDF to yield a better compromise between hp-adaptivity and the efficiency of the modified Newton method whose convergence criteria (to control the iterative error) are based on the chosen error tolerance (to control the discretization error). In practice, the main advantage of the procedure is that the user only has to select the desired accuracy tolerance for a given problem at hand. It frees him from any further considerations about the time integration procedure.

Numerical experiments on flows over fixed and impulsively rotating circular cylinders at several low Reynolds numbers demonstrate the efficiency and reliability of the proposed approach.

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