Model Predictive Control for Probability Density Functions

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Abstract: The evolution of probability density functions for stochastic processes modeled by ordinary Itô stochastic differential equations is determined by the Fokker-Planck equation, a second order parabolic PDE. Thus, the problem of optimally controlling the probability density function can be reformulated as a control problem for this PDE.

In this talk, we apply a Model Predictive Control scheme to the problem of tracking the solution of the Fokker-Planck equation, i.e., the desired probability density function, over a fixed time interval. The control is a piecewise constant function in time and may also depend on space. Several tracking objectives are employed to encompass a variety of possible applications. Each cost functional is presented along with the corresponding optimality system which is characterized by an adjoint PDE. Furthermore, the dependence of the MPC algorithm on the prediction horizon, the regularization parameter and the sampling time is analyzed. The computed optimal control of the probability density function is then applied to the stochastic process. In the case of space-dependent control this translates into a feedback loop.

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