Properties of Singular Linear-quadratic Control Problem and Euler-Poisson Equations

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Abstract: Consider the quadratic functional

$$I(u) = \int_{\alpha}^{\beta} \left[\left\langle \tilde{A}u, u \right\rangle + 2 \left\langle \tilde{B}u, x \right\rangle + \left\langle \tilde{C}x, x \right\rangle \right] dt, \tag{6}$$

under constraints

$$A\dot{x} + Bx = Cu, \ x(\alpha) = 0, \ t \in T = [\alpha, \ \beta], \tag{7}$$

where $\tilde{A} \equiv \tilde{A}(t)$ is a $(m \times m)$-matrix, $\tilde{B} \equiv \tilde{B}(t)$ is a $(n \times m)$-matrix, $\tilde{C} \equiv \tilde{C}(t)$ is a $(n \times n)$-matrix, $A \equiv A(t), \ B \equiv B(t)$ are $(n \times n)$-matrices, $C \equiv C(t)$ is a $(n \times m)$-matrix, $x \equiv x(t)$ is a n-dimensional vector-function from $\mathbf{C}^1(T)$, $u \equiv u(t)$ is a m-dimensional vector-function, $\langle . \, , . \rangle$ is a scalar product in Euclidean space. It is supposed that

$$\det \tilde{A}(t) = 0, \ \det A(t) = 0 \ \forall t \in T,$$

and the initial data of (6), (7) is sufficiently smooth for subsequent reasoning. Matrices $\tilde{A}, \ \tilde{C}$ in (6) are symmetric which does not reduce the generality of the problem statement.

This talk addresses the structure of general solutions for such systems and some of their properties. Thereupon positiveness conditions for the objective functional have been obtained, conditions for small deviations from the starting point have been established for small values of the functional. This problem has been previously addressed in [1]. We suggest an approach which does not require the construction of Pontryagin's functional.

References

[1] V. F. Chistyakov, M. Peshich. On the properties of the identically singular Lagrange problem//Automation and Remote Control. 2009. Vol. 70, Issue 1, pp. 74–91.

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