Analysis of the Problem of Natural Convection with Radiative Heat Transfer

A. Yu. Chebotarev[1] **and A. E. Kovtanyuk**[2]

Abstract: Stationary process of free convection of a viscous incompressible fluid with radiation in a bounded domain Ω with boundary Γ is modeled by the following system, which uses a diffusion P_1 approximation for the radiative transfer equation.

$$-a\Delta\theta + \mathbf{v}\cdot\nabla\theta + b\kappa_a\theta^4 = b\kappa_a\varphi, \quad -\alpha\Delta\varphi + \kappa_a\varphi = \kappa_a\theta^4, \tag{3}$$

$$-\nu\Delta\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} + \beta\theta\mathbf{g} = -\nabla p, \ \operatorname{div}\mathbf{v} = 0. \tag{4}$$

Here, θ is the normalized temperature, φ the normalized radiation intensity averaged over all directions, $\mathbf{v}$ is the velocity field and p is the flow pressure, $\mathbf{g}$ is an acceleration of gravity. Through a, ν, β designated constant coefficients of thermal diffusivity, kinematic viscosity and thermal expansion. Parameters $\alpha > 0$, $b > 0$, and the absorption coefficient κ_a describes the radiation-thermal properties of the medium. To the equations (3)-(4) we add the boundary value conditions

$$\theta|_\Gamma = \Theta_0, \quad \alpha\partial_n\varphi + \gamma(\varphi - \Theta_0^4)|_\Gamma = 0, \quad \mathbf{v}|_\Gamma = \mathbf{v}_0. \tag{5}$$

Analysis of complex heat transfer in scattering media with reflecting boundaries is important for applications. A lot of work is connected with the numerical simulation of complex heat transfer processes in continuous media. At the same time, few papers devoted to the theoretical analysis of the corresponding boundary value problems, that allows you to assess the adequacy of the models of radiative heat transfer.

The main result of this work is to obtain new a priori estimates of temperature and radiation intensity in the space L^∞, which is possible to prove the solvability of the problem. It is shown that the class of weak solutions is homeomorphic to finite-dimensional compact. It is proved that the solution is unique, if the viscosity and thermal diffusivity are sufficiently large.

The research was supported by the Ministry of Education and Science of Russian Federation (project 14.Y26.31.0003).

[1,2] Department of Informatics, Mathematical and Computer Modeling
Far Eastern Federal University
Vladivostok, Russia
chebotarev.ayu@dvfu.ru, kovtanyuk.ae@dvfu.ru