Multiple Shooting Approach for Computing Geometric Shortest Paths on Polytopes

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Abstract: We present a new approach called multiple shooting for computing shortest paths from point p to point q (z-coordinate of p is greater than that of q) on the surface of a polytope. Firstly, the polytope is divided into subpolytopes by parallel cutting slices through the vertices of the polytope between p and q in terms of z-coordinate, a shooting point is initialized on each cutting slice. The shortest paths between two consecutive shooting points on every subpolytope are determined and then combined into a path from p to q. Secondly, since the first segments of these shortest paths incident at each shooting point are collinear, a straight condition is constructed to check if the obtained path is an approximate shortest path. Finally, a update of the shooting points is performed due to the straight condition such that the length of the path formed by shooting points is non-increasing. We then obtain an approximate shortest path. Our method is similar to the geometrical idea of the well-known multiple shooting method for solving the ODE-boundary value problems.

Unlike some methods for computing shortest paths on polytopes, such as Pham-Trong et al.'s (*Numerical Algorithms* 26, 2001) and Xin and Wang's (*Computer-Aided Design* 39, 2007), which also generate a set of sequences of faces of polytopes, our method updates a sequence of faces only at the shooting points, where the angles of the path are not satisfied with the straight condition, but not all vertices of the current path. In addition, our method does not use Steiner point technique and graph tools on the entire the surface that Agarwal et al.'s algorithms (*Journal of the ACM* 44, 1997 and *Algorithmica* 33, 2002) and the others rely on.

By the same manner, computing shortest descending path between two points on a terrain, posed by de Berg and van Kreveld (*Algorithmica* 18, 1997) is also dealt with (for the case of convex terrains).

The algorithms are implemented in C++ using CGAL. Numerical experiments compared with the Agarwal et al.'s algorithm on convex polytopes show that the multiple shooting approach is better on the accurate construction of the shortest path.

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