Asymptotic Solution of Singularly Perturbed Optimal Problem in the case of Intersection of Solutions of Degenerate Problem

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Abstract: We will consider the following problem

$$P_\varepsilon\colon\ J_\varepsilon(u) = \int_{t_0}^{T} F(x,y,u,t,\varepsilon)dt \to \min_u,$$

$$\frac{dx}{dt} = f(x,y,u,t,\varepsilon), \quad \varepsilon\frac{dy}{dt} = g(x,y,t,\varepsilon),$$

$$x(t_0,\varepsilon) = x^0, \quad y(t_0,\varepsilon) = y^0.$$

Here $x = x(t,\varepsilon) \in R^l$, $y = y(t,\varepsilon) \in R^m$,$u = u(t,\varepsilon) \in R^r$, $t \in [t_0,T]$; $\varepsilon \geq 0$ is a small parameter; all functions F, f, g are sufficiently smooth in their domain where all derivatives are continuous.

We construct the asymptotical expansion of solution of the problem P_ε with the crucial assumption is that the algebraic equation

$$g(x,y,t,0) = 0$$

has two solutions $y = \varphi_1(x,t)$ and $y = \varphi_2(x,t)$ which intersect at a point $t_c \in (t_0,T)$ and exchange their stability.

For constructing the asymptotic solution of the problem P_ε we use the accordingly values $y = \varphi_1(x,t)$ and $y = \varphi_2(x,t)$ on each time intervals $[t_0,t_c]$ and $[t_c,T]$. The postulated asymptotic expansion with boundary functions will be substituted into the original problem P_ε and then four types of optimal control problems are defined to get the asymptotic terms. We show that a minimized functional is non-increasing if higher order approximations to the optimal control are used. The estimates of the proximity of an approximate asymptotic solutions to the exact one are given.

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