Regularization Methods for Nonexpansive Semigroups on Hilbert Spaces

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Abstract: Let H be a real Hilbert space with inner product $\langle ., . \rangle$ and norm $\|.\|$. In this report, we introduce a regularization scheme based on Browder-Tikhonov regularization method and two different combinations of the proposed regularization method with iterative process and inertial proximal point algorithm for the following problem: Find an element $p \in \mathcal{F}$ such that

$$\|x_* - p\| = \min_{y \in \mathcal{F}} \|x_* - y\|$$

where x_* is an element in H such that $x_* \notin \mathcal{F}$ and $\mathcal{F}$ is the set of common fixed points of a nonexpansive semigroup $\{T(s) : \ s > 0\}$ on H.

The first method is Browder-Tikhonov regularization: find $x_n \in H$ such that

$$A^C(t_n)x_n + \alpha_n(x_n - x_*) = 0, \quad A^C(t) = I - T(t)P_C,$$

where I denotes the identity mapping in H and $\{t_n\}$, $\{\alpha_n\}$ are two sequences of positive real numbers tending to zero as $n \to \infty$. The second scheme is constructed based on a combination of the proximal point method with the regularization one, named the regularization proximal point algorithm. The idea used in this research is to generate an approximation sequence $\{z_n\}$ for the stated problem by the following equation:

$$c_n[A^C(t_n)z_{n+1} + \alpha_n(z_{n+1} - x_*)] + z_{n+1} = z_n, \quad n \geq 0, z_0 \in H,$$

where $\{c_n\}$, $\{t_n\}$ and $\{\alpha_n\}$ are three sequences of real positive numbers such that $\{t_n\}$ and $\{\alpha_n\}$ tend to zero as $n \to \infty$. The third one is established by combining the considered regularization method with an explicit iterative process. Start with a given point $w_0 \in H$ and define a sequence $\{w_n\}$ iteratively by the following rule:

$$w_{n+1} = w_n - \beta_n[A^C(t_n)w_n + \alpha_n(w_n - x_*)], \quad n \geq 0,$$

where $\{\beta_n\}$ is also a sequence of positive real numbers satisfying some control condition. Some applications of the proposed methods are also considered.

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