Automatic Spectral Collocation for Higher-Order ODE State Estimation

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Abstract: The computational problem we address in this paper is the estimation of a state function representing a solution of a higher-order linear ODE (i.e. the process equation) within a specified time interval. In place of unknown boundary conditions we have a set of noisy measurements. Measurement equations are provided and measurement errors are assumed normally distributed with zero mean and known variance. Each measurement equation links a known measurement function with an unknown state function and its derivatives. Linear differential operators for process and measurement equations and their adjoint operators are formulated. To avoid computational problems with large condition numbers that are present for discretized higher-order differential operators, we reformulate the equations into the integral equations [1, 2] and use integral operators and their adjoints instead. The standard approach in estimation based on maximum likelihood has been used. The process equation is included as the equality constrain. By setting the first variation of the augmented Lagrangian functional to zero we obtain the linear operator equation with a block structure to solve. Matrix-free iterative methods are good choice here, e.g. GMRES. The Chebyshev spectral collocation with automatic selection of discretization grid resolution has been used for solving matrix-vector products required by a functional form of the GMRES iteration [2]. It should be noted that high computation efficiency is achieved in this matrix-vector product by using efficient implementation of the Discrete Cosine Transform to compute indefinite integrals. The paper also shows how to solve nonlinear state estimation problems by using functional Newton iterations and the proposed technique as the internal linear equation solver. The method is implemented using chebfun software system [3]. New objects for adjoint differential and integral operators are integrated into the chebfun system.

References

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