Adaptive Finite Element Discretizations in Structural Optimization

W. Wollner[1]

Abstract: In this talk we will consider an optimization problem of the form

$$\min J(q,u)$$

subject to control constraints $q \in Q^{ad}$. Where Q^{ad} is a closed convex subset of a Banach space Q. Furthermore the state $u \in V$ and the control $q \in Q^{ad}$ are coupled by the partial differential equation

$$a(q,u)(\phi) = 0 \quad \forall \phi \in V.$$

As it is well known that the effort for the optimization is directly linked to the number of unknowns we will derive an a posteriori error estimate in order to drive local mesh refinement with respect to a given target quantity.

As a prototypical example we consider the well known compliance minimization of a variable thickness sheet, e.g. given a domain $\Omega \subset \mathbb{R}^2$, we consider

$$\min_{q \in L^2, u \in H^1_D} \; l(u)$$

subject to the constraints

$$(q\,\sigma(\nabla u), \nabla\phi) = l(\phi) \quad \forall\, \phi \in H^1_D(\Omega;\mathbb{R}^2),$$

$$0 < q_{\min} \le q \le q_{\max},$$

$$\int_\Omega q \le V_{\max},$$

where $H^1_D(\Omega;\mathbb{R}^2)$ denotes the usual H^1-Sobolev space with certain Dirichlet boundary conditions, and $\sigma(\nabla u)$ denotes the usual (linear) Lamé-Navier stress tensor.

Finally we will give an outlook to possible extensions.

[1] Department of Mathematics
University of Hamburg
Bundesstr. 55, 20146 Hamburg, Germany
winnifried.wollner@math.uni-hamburg.de