Global Error Bounds for Some Classes of Multivariate Polynomials

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Abstract: Let $f : \mathbb{R}^n \to \mathbb{R}$ be a polynomial

$$S := \{x \in \mathbb{R}^n : f(x) \leq 0\}.$$

f has a global Hölderian error bounds if there exist $c > 0, \alpha > 0, \beta > 0$ such that

$$d(x, S) \leq c([f(x)]_+^\alpha + [f(x)]_+^\beta)$$

for all $x \in \mathbb{R}^n$, where $d(x, S)$ denotes the Euclidean distance between x and S, and $[f(x)]_+ = max\{0, f(x)\}$.

First, we give a criterion for the existence of a global Hölderian error bound. Next, we prove that if the Palais-Smalle condition holds at each nonnegative value then f satisfies this criterion. Consequently, every tame polynomial in the sense of Broughton [Invent. Math. 92 (1988), pp. 217-241] also satisfies this criterion. Further, we prove that if f is convernient and non-degenerate with respect to its Newton boundary at infinity in the sense of Kouchnirenko [Invent. Math. 32 (1976), pp. 1-32] then it is tame. Finally, it will be demonstrated that in the variety of polynomial with a given Newton boundary at infinity the non-degenerate polynomials form a dens, open subset.

Altogether, these results show that global Hölderian error bounds hold for a large class of polynomials.

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