An Improved Direct Multiple Shooting Approach Combined with Collocation and Parallel Computing to Handle Path Constraints in Dynamic Nonlinear Optimization

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Abstract: We present an improved solution strategy to solve a class of dynamic nonlinear optimization problems governed by differential-algebraic equations (DAEs) with path constraints on both control and state variables. The proposed approach is an extension of the work by Tamimi and Li (2010) which combines the direct multiple shooting and collocation method for solving such optimization problems. The extension lies in the consideration of state path constraints inside each shooting interval, i.e. we force state inequalities to be satisfied at the inner collocation points. This leads to an improved feasibility of the solution compared to previous methods where path constraints are considered only at grid points of the shooting intervals. In addition, we use parallel computing for each shooting interval to enhance the computational efficiency.

We consider the following dynamic nonlinear optimization problem:

$$\min_{\mathbf{u}(t),\mathbf{x}(t),\mathbf{y}(t)} \quad \varphi\big(\mathbf{x}(t),\mathbf{y}(t),\mathbf{u}(t)\big) \tag{1}$$

$$s.t. \quad \dot{\mathbf{x}}(t) = \mathbf{F}(\mathbf{x}(t),\mathbf{y}(t),\mathbf{u}(t));\ \mathbf{x}_0 \ = \ \mathbf{x}(t_0) \tag{2}$$

$$\mathbf{G}(\mathbf{x}(t),\mathbf{y}(t),\mathbf{u}(t)) \ = \ \mathbf{0} \tag{3}$$

$$\mathbf{u}_l \le \mathbf{u}(t) \le \mathbf{u}_u;\ \mathbf{x}_l \le \mathbf{x}(t) \le \mathbf{x}_u;\ \mathbf{y}_l \le \mathbf{y}(t) \le \mathbf{y}_u \tag{4}$$

where φ is a scalar objective function, $\mathbf{x}(t) \in R^{NX}$, $\mathbf{y}(t) \in R^{NY}$, $\mathbf{u}(t) \in R^{NU}$ are differential and algebraic states, and control variables, respectively. Eq. (4) presents path constraints on control and state variables. By using direct multiple shooting (DMS) with piecewise constant parameterization of control variables, we convert the problem into a NLP problem. Then we use collocation on finite elements to solve the DAEs in each shooting interval by the Newton method and compute gradients with the automatic differentiation. The advantage of DMS is that functions in each shooting interval depend only on their local variables so that we can evaluate them independently by parallel computing.

We use the interior point method to solve the NLP problem due to its powerful capability of dealing with inequalities. The parallel computing is implemented in the parallel programming framework of message passing interface (MPI). The performance of the proposed approach will be presented by using optimal control and parameter estimation examples.

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