Adaptive Multilevel Optimization with Reduced Order Models for PDE-constrained Problems

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Abstract: We present an adaptive multilevel generalized SQP-method for optimal control problems governed by nonlinear time-dependent PDEs with control constraints. For such problems, standard discretization techniques require in general many degrees of freedom for the optimization process and a good resolution of the optimal solution which is very time consuming. To overcome this difficulty we approach this problem by optimizing in suitable subspaces in the spatial dimension with much fewer degrees of freedom. These subspaces are then adapted towards the optimal solution. This principle is realized by optimizing with reduced order models on adaptively refined discretizations.

During the optimization iterations the algorithm generates a hierarchy of adaptively refined discretizations. The discretized problems are then each approximated by an adaptively generated sequence of reduced order models. The adaptive refinement strategies are based on a posteriori error estimators for the original and the discretized PDE, the adjoint PDE and a criticality measure. In the presented optimization framework we follow the 'first optimize, then discretize' approach which allows to use any kind of PDE solvers provided by the user. The resulting inexactness in the current discretization is controlled within the algorithm. Consequently, significantly fewer degrees of freedom are necessary. Moreover, only few PDE simulations of the discretized problems are needed where we start on coarse meshes. This promises a substantial save of computational time while the accuracy of the optimization result is controlled by the error estimators. Convergence to a first-order optimality point is proven.

In our numerical examples we construct the reduced order models by the proper orthogonal decomposition (POD) method from snapshots of the state and adjoint state. The effort for the nonlinearities of the resulting POD Galerkin discretization is additionally reduced by the discrete empirical interpolation method (DEIM) recently proposed by Chaturantabut and Sorensen. Numerical results for test problems and a 3D glass cooling application are presented.

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