Smoothing Techniques and Dual Decomposition in Structured Large-Scale Optimization and Applications

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Abstract: In the first part of this work, we present some recently novel methods for solving separable large-scale convex optimization problems of the form:

$$\begin{array}{lll}\min\limits_{x} & \sum_{i=1}^{M} f_i(x_i) & \text{(separable objective function)}\\ \text{s.t.} & \sum_{i=1}^{M} A_i x_i = b, & \text{(linear coupling constraints)}\\ & x_i \in X_i,\ i = 1, \ldots, M, & \text{(local convex constraints)}\end{array} \qquad \text{(CP)}$$

where $f_i : \mathbb{R}^{n_i} \to \mathbb{R}$ is convex, $X_i \in \mathbb{R}^{n_i}$ is nonempty, closed bounded and convex, $A_i \in \mathbb{R}^{m\times n_i}$ for $i = 1, \ldots, M$ and $b \in \mathbb{R}^m$. This formulation covers many application problems arising in linear programming, separable quadratic programming, network optimization and multistage stochastic convex optimization problems.

The main idea of the proposed methods is based on a combination of Lagrangian dual decomposition, smoothing techniques and structured convex optimization methods. By applying the Lagrangian relaxation technique, the dual function of the original problem (CP) is decomposed into the smaller dual component problems. Then, they are smoothened by smoothing techniques via proximity functions and self-concordant barrier functions. Fast-gradient schemes as well as path-following methods are employed to solve the smoothed dual problem. The algorithms that we propose possess many advantages such as fast convergence and numerical stability compared to previously known methods.

In the second part, we propose some algorithms to solve nonconvex separable optimization problems. These methods are a combination of the penalty and the augmented Lagrangian function techniques and the dual decomposition. Applications in distributed linear and nonlinear model predictive control, network optimization and multistage stochastic optimization are presented. Numerical results show the superior of the proposed algorithms to the previously known methods.

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