Numerical Approach to the Non-stationary Seepage in the Flow through an Absorbing Medium

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Abstract: We are concerned with the dynamical behaviour of non-stationary seepage in the flow through an absorbing medium. In particular, it is expected that such a seepage exhibits *support splitting and merging phenomena*, which is caused by the interaction between the nonlinear diffusion and the penetration of the fluid from the boundary on which the flowing tide and the ebbing tide occur. Here the support means the region where the fluid exists.

The model equation which describes such phenomena is written in the form of the nonlinear initial-boundary value problem:

$$\begin{cases} v_t(x,y,t) = \Delta v^m - cv^p & \text{in } \Omega \times (0,\infty), \\ v(x,y,t) = \psi(x,y,t) & \text{in } S_t = \partial\Omega \times (0,\infty), \\ v(x,y,0) = v^0(x,y) & \text{in } \Omega, \end{cases}$$

where v denotes the density of the fluid, $m > 1$, $0 < p < 1$, $c > 0$, $m + p = 2$ and $v^0(x,y)(\geq 0)$. Kersner proved the appearance of *support splitting phenomena* ([1]) in $\mathbf{R}^1$, but he did not show that the *support merging phenomena* appear after the support splits. We tried the numerical computation by using our scheme ([2]), and found the *support splitting and merging phenomena*.

In this talk, we show the relation between the length of interval $\Omega = (-L,\ L)$ and the boundary conditions $\psi(\pm L, t)$ in $\mathbf{R}^1$ under which such phenomena appear. Thus the support splits for a large L, but never splits otherwise. Moreover, we demonstrate the behaviour of the numerical support in $\mathbf{R}^2$.

References

[1] R. Kersner, Degenerate parabolic equations with general nonlinearities, *Nonlinear Anal.*, **4** (1980), pp. 1043–1062.

[2] T. Nakaki and K. Tomoeda, A finite difference scheme for some nonlinear diffusion equations in an absorbing medium: support splitting phenomena, *SIAM J. Numer. Anal.*, **40** (2002), pp. 945–964.

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