Numerical Treatment in Resonant Regime for Shallow Water Equations with Variable Topography

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Abstract: We are interested in numerical approximations in the resonant regime of solutions of the shallow water equations with variable topography

$$\begin{aligned}&\partial_t h + \partial_x(hu) = 0,\\ &\partial_t(hu) + \partial_x\Big(h(u^2 + \tfrac{gh}{2})\Big) + gh\,\partial_x a = 0, \quad x \in \mathbb{R}, t > 0,\end{aligned}$$

where h is the height of the water from the bottom to the surface, u is the water velocity, g is the gravity constant, and $a = a(x), x \in \mathbb{R}$, is the height of the bottom from a given level.

Many authors have presented well-balanced numerical schemes for the above shallow water equations. These schemes have been shown to work well when data are in a strictly hyperbolic domain. However, when data are on both sides of the resonance surface on which the system fails to be strictly hyperbolic, existing schemes often give unsatisfactory results. In this work, we improve the well-balanced scheme constructed in our earlier work by introducing a corrector in the computing algorithm that selects the admissible equilibrium states. The corrector then could enable the computing algorithm to take exactly the admissible equilibrium state. We then present numerical tests which show that our modified well-balanced can approximate the exact solution for any initial data.

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