On a Lavrentiev Finite Element Solution of the Data Completion problem

D. D. Thang[1]

Abstract: The Data Completion problem of recovering data on the unreachable boundary associated with the Cauchy problem: *find u such that*

$$\begin{aligned}(-\Delta)u &= 0 \qquad \text{in } \Omega,\\ u=g,\ \partial_{\mathbf{n}}u &= \varphi \qquad \text{on } \Gamma_C,\\ u &= ? \qquad \text{on } \Gamma_I.\end{aligned}$$

We are led to the study of the following Steklov-Poincaré problem: *find $\lambda \in H^{1/2}(\Gamma_I)$ such that*

$$s(\lambda,\mu)=\ell(\mu), \qquad \forall \mu \in H^{1/2}(\Gamma_I).$$

A conformal finite element discretization related to the problem (e.g. the Galerkin finite element discretization) leads to the discrete problem, which is severely ill-posed, as follows: *find $\lambda_h \in D_h^I$ such that*

$$s(\lambda_h,\mu_h)=\ell(\mu_h), \qquad \forall \mu \in D_h^I.$$

The Lavrentiev regularization of the discrete problem reads as: *find $\lambda_{\alpha,h} \in D_h^I$ such that*

$$\alpha s_D(\lambda_{\alpha,h},\mu_h)+s(\lambda_{\alpha,h},\mu_h)=\ell(\mu_h), \qquad \forall \mu \in D_h^I.$$

The element $\alpha s_D(\lambda_{\alpha,h},\mu_h)$ is to restore the coerciveness of the bilinear form. A convergence analysis is given in the case where Cauchy data (g,φ) are compatible (the exact case), and incompatible (the perturbation case) $(g^\delta,\varphi^\delta)$. Under this analysis, an *a-priori* choice of the regularization parameter α is given, so that the sequence of perturbed approximation solution $(\lambda^\delta_{\alpha,h})_\alpha$ converges towards the exact solution λ conditionally. Furthermore, an *a-posteriori* stopping criteria of the iteration procedure is established by using the Morozov Discrepancy principle, i.e. we stop the iteration at some α depending only on the perturbation Cauchy data $(g^\delta,\varphi^\delta)$ such that the iterative sequence can be considered as an appropriate approximation of the exact solution, even though it blows up generally. What is more, under some more regularity assumption on the solution, we establish some convergence rates of the approximation towards the exact solution in both *a-priori* and *a-posteriori* case. Some numerical experiences are presented to verify our theoretical analysis.

[1] Faculty of Mathematics, Mechanics and Informatics
Hanoi University of Science Vietnam National University - Hanoi
334 Nguyen Trai Street, Hanoi, Vietnam
thangdd@vnu.vn, duducthang@gmail.com