

# Numerical Simulations of Two-fluid Flow Problems by a Galerkin-characteristic Finite Element Scheme

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**Abstract:** Galerkin-characteristics finite element approximation is a powerful numerical method for flow problems (see [1, 2] and the references therein for recent results of the Galerkin-characteristics finite element method). Recently we have developed an energy-stable Galerkin-characteristics scheme to two-fluid flow problems with surface tension, where each flow is governed by the Navier-Stokes equations. We have approximated the convection terms of the velocity and the density by a Galerkin-characteristics finite element method. A careful device of the method of characteristics maintains the energy-stability property of the scheme. Thus the obtained scheme has the advantages of the method of characteristics, namely, robustness for high Reynolds number problems and symmetricity of the resultant matrix, as well as the energy-stability. The computation time can be reduced to about 40% of that of the previous scheme [3, 4]. Here we report some numerical simulation results of two-fluid flow problems of rising bubbles and falling droplets using this scheme. Those results show the robustness, the efficiency, and the applicability of the scheme.

## References

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