Recursive Optimization Methods for Inverse Obstacle Scattering Problems

M. Sini[1] **and** **N. T. Thanh**[1]

Abstract: We investigate recursive optimization algorithms for an inverse acoustic obstacle scattering problem. In the scattering of time-harmonic incident plane waves by a sound-soft obstacle $D \in \mathbb{R}^2$, the scattered wave is given by the Helmholtz equation

$$\Delta u^s(x) + k^2 u^s(x) = 0, \ x \in \mathbb{R}^2 \setminus \bar{D},$$

with the boundary condition $u^s(x) = u^i(x), \ x \in \partial D$, and the Sommerfeld radiation condition $\lim_{|x| \to \infty} \sqrt{|x|}\,[\partial u^s(x)/\partial |x| - iku^s(x)] = 0$. Here k is the wavenumber (or frequency), $u^i(x) := e^{ikx\cdot\theta}$ is the incident plane wave with $\theta \in S^1$ - the unit circle. The associated *far field pattern* $u^\infty(\hat{x})$ is defined from the asymptotic expansion $u^s(x) = \frac{e^{ik|x|}}{\sqrt{|x|}} u^\infty(\hat{x}) + O(|x|^{-3/2}), \ |x| \to \infty$, where $\hat{x} := x/|x|$.

The inverse problem to be tackled is *to reconstruct the obstacle D from measured far field pattern $u^\infty(\hat{x}, k), \ \hat{x} \in S^1$, for one incident direction $\theta \in S^1$ and multiple wavenumbers k in the interval $[a, b]$.*

To solve this inverse problem, we make use of recursive optimization algorithms. The idea is to reconstruct a rough approximation of the obstacle at the lowest frequency using the least-squares approach. Then, the reconstruction is recursively improved using higher frequencies. This approach enables us to obtain an accurate reconstruction without requiring a good initial guess.

The analysis is divided into three steps. In the first step, we obtain a quantitative estimate of the set of local convexity of the objective functional at a fixed frequency. Our analysis shows that, the size of this set is inversely proportional to the used frequency. Consequently, if the obstacle is contained in a given domain, the lowest frequency should be chosen small enough so that this set of convexity covers the given domain and any shape in this domain can be used as an initial guess. This choice of the lowest frequency helps avoiding the need of a good initial guess. However, due to the lack of good stability at low frequencies, we can only expect a rough reconstruction. To enhance the accuracy, we use the recursive optimization algorithms at higher frequencies. The second step is devoted to the convergence rate of the recursive optimization algorithms. In the third step, we justify a conditional asymptotic Lipschitz stability estimate on the parts of the obstacle's boundary illuminated by the incident wave in the high frequency regime. This result explains why we can obtain an accurate reconstruction of the illuminated part of the obstacle.

[1] Johann Radon for Comp. & Appl. Math. (RICAM)
Austrian Academy of Sciences
Altenbergerstrasse 69, A-4040 Linz, Austria
{*mourad.sini, trung-thanh.nguyen*}*@oeaw.ac.at*