Sparse Tensor Discretizations of PDEs with Random Input

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Abstract: We address the numerical analysis and the computer implementation of solution algorithms for partial differential equations (PDEs) with random field inputs, being either diffusion equations with stochastic coefficients, or conservations laws with random initial data or random flux.

We present mathematical results which give sufficient conditions under which reformulation of such problems as infinite-dimensional, parametric deterministic PDE reformulations of such problems are possible. New results on regularity of the reformulated problems are presented which imply that deterministic, adaptive approximation schemes of stochastic Galerkin resp. stochastic collocation type can give convergence rates which are superior to rates afforded by Monte-Carlo (MC) methods. In hyperbolic conservation laws, a new smoothing mechanism for statistical moments of random entropy solutions is described.

We will also report on recent advances in the analysis and the massively parallel implementation of Multi-Level MC methods and Multi-Level Quasi Monte-Carlo Methods, for nonlinear PDEs with random inputs.

The results presented in this talk are based on joint work with A. Cohen (Paris), R. DeVore(Texas A&M) , C.J. Gittelson (Purdue), V.H. Hoang (NTU Singapore), S. Mishra (ETH), J. Šukys(ETH) and S. Tokareva (ETH).

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