New Jump Conditions in State-constrained Optimal Control for a Coupled ODE-PDE System

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Abstract: The motivation for the theoretical investigation of the coupled ODE-PDE optimal control problem of this talk comes from a minimum-fuel trajectory optimization problem for a hypersonic passenger aircraft. There, the main part of the model is given by an aerothermic heat constraint for the thermal protection system (TPS) which is essential at hypersonic speeds. This trajectory optimization problem is modelled by a system of nonlinear ordinary differential equations (ODE) for the flight dynamics where a state constraint on the temperature of the TPS constitutes the most important inequality constraint. For it a heat equation with non-linear boundary conditions and coefficients depending on the ODE state variables and controls must be taken into account.

Since the complexity of this problems prevents any deaper mathematical analysis, a class of academic problems called *hypersonic rocket car problems* has been invented. One type of problems consist of a linear second order ODE control system and a heat equation with a source term so that the system exhibits the same highly nonlinear coupling structure as the hypersonic flight problem. This problem is simple enough to do some mathematical analysis. We will particularly focus on new jump conditions arising among the necessary conditions and their numerical exploitation resp. verification [3]. In contrast to [1], [2],[4], where either pure ODE or pure PDE versions of the hypersonic rocket car problems have been studied, we treat that problem here as an ODE-PDE, resp. PDAE optimal control problem, i. e. for a system of partial differential algebraic equations [3].

References

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