Optimal Linear Approximations for MINLP Problems

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Abstract: Beale & Forrest (1976) presented the idea of linear approximations to compute a global minimum of non-convex nonlinear functions. They introduced special ordered sets of type 2 (SOS-2) and efficient branching schemes to exploit this structure. Since then, various contributions elaborated on the usage of SOS-2. What all of them have in common is that they use a given set of breakpoints.

In this paper, for functions depending on one or two variables, we systematically construct optimal breakpoint systems subject to the condition that the linear approximation never deviates more than a given ϵ-tolerance from the original function over a given domain. The optimization problem of computing the minimal number of breakpoints satisfying the ϵ-tolerance leads to semi-infinite problems. We introduce several discretization schemes and algorithms, computing linear approximator, underestimator and overestimator systems with ϵ-tolerance.

The optimal breakpoints for the nonlinear functions are then used in the MILP replacement of the original NLP or MINLP problem, where nonlinear equations are replaced by two inequalities relaxed by ϵ. Due to the ϵ-limited discretization error and the minimal number of breakpoints, the solution of the MILP problem can be obtained in reasonable time and serves a good approximation to the global optimum, which can be fed into a local NLP solver for the final refinement. Possible extensions to higher dimensions are discussed as well.

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