

The Subtour LP for the Traveling Salesman Problem

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Abstract: The traveling salesman problem (TSP) is the most famous problem in discrete optimization. Given n cities and the costs c_{ij} for traveling from city i to city j for all i, j , the goal of the problem is to find the least expensive tour that visits each city exactly once and returns to its starting point. We consider cases in which the costs are symmetric and obey the triangle inequality. In 1954, Dantzig, Fulkerson, and Johnson introduced a linear programming relaxation of the TSP now known as the subtour LP, and used it to find the optimal solution to a 49-city instance. Ever since then, the subtour LP has been used extensively to find optimal solutions to TSP instances, and it is known to give extremely good lower bounds on the length of an optimal tour.

Nevertheless, the quality of the subtour LP bound is poorly understood from a theoretical point of view. For 30 years it has been known that it is at least $2/3$ times the length of an optimal tour for all instances of the problem, and it is known that there are instances such that it is at most $3/4$ times the length of an optimal tour, but no progress has been made in 30 years in tightening these bounds.

We will review some of the results that are known about the subtour LP, and give some new results that refine our understanding in some cases. In particular, we resolve a conjecture of Boyd and Carr about the ratio of an optimal 2-matching to the subtour LP bound in the worst case. We also begin a study of the subtour LP bound for the extremely simple case in which all costs c_{ij} are either 1 or 2. For these instances we can show that the subtour LP is always strictly better than $3/4$ times the length of an optimal tour.

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