Hilbert Space Methods for the Solution of Infinite Horizon Optimal Control Problems

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Abstract: Still at the beginning of the previous century the optimal control problems with infinite horizon became very important with regards to applications in economics, where an infinite horizon seems to be a very natural phenomenon. These problems were treated by many authors and various necessary, sufficient as well as transversality conditions were obtained. The problem we consider in this talk is formulated as optimization problem in Hilbert Spaces. It reads as follows: Minimize the functional

$$J(x,u)=\int\limits_0^\infty r(t,x(t),u(t))\nu(t)dt$$

subject to all pairs $(x,u)\in W_2^1(\mathbb{R}^+,\nu)\times L_2(\mathbb{R}^+,\nu)$, satisfying state equations

$$\dot{x}(t)=f(t,x(t),u(t))\,,$$

control restrictions

$$u(t)\in U,\quad U\in Comp\,(R^r)\setminus\{\,\emptyset\,\},$$

and initial conditions

$$x(0)=x_0.$$

The integral in the functional J is understood in Lebesgue sense. The remarkable on this statement is the choice of Weighted Sobolev- and Weighted Lebesgue spaces as state and control spaces respectively. The function ν is a density function. These considerations give us the possibility to extend the admissible set and simultaneously to be sure that the adjoint variable belongs to a Hilbert space.

For the class of problems proposed, we can proof an existence result as well as Pontryagins Maximum Principle. An Ritz – method is the developed to construct a numerical scheme for the solution of the problem.

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